N-Factor Satisfactory Marriage Problem with Varying Weight and Unequal Preference List

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ABSTRACT

Gale and Shapley proposed a GS algorithm to solve Marriage Problem. This algorithm results a man optimal or women optimal stable matching. In this paper a new algorithm called SMA is applied to N-Factor Marriage Problem with varying weight and Unequal Preference list. The findings were discussed and illustrated with real life examples.

Keywords: Optimal matching, Preference Value, Satisfactory value matrix, Satisfactory Level

1. INTRODUCTION

The Stable Marriage Problem(SM) was introduced by Gale and Shapley (1962) [1].An SM instance consists of two finite equal sized set of members, called men and women. Each man m_i ($1 \le i \le n$, n is the number of men) prefer women in strict order, forming his own preference list. Similarly, each woman w_j ($1 \le j \le n$) prefer men in strict order, forming her own preference list [2]. A Stable Marriage is a one to one matching of the men with the women such that there is no man-woman pair that prefers each other over their present mates. Gale and Shapley describe a simple algorithm (GS algorithm) [3] which, form stable matching based on the given arbitrary preference list.

Gale and Shapley showed that at least one stable matching exists for every SM [4]. However, in general there are many different stable matching for a single instance, and the GS algorithm finds only one of them (man-optimal or woman optimal) with an extreme property. In the man-optimal stable matching, each man is matched with his best possible partner, while each woman gets her worst possible (or if we exchange the role of men and women, the resulting matching is woman optimal) [5]. Hence, it is natural to try to obtain a matching which is not only stable but also "good" in some criterion.

A Satisfactory Marriage problem was solved in [6] by considering one factor in the preference list. A Marriage problem by considering N-factors was solved in [7].The weightage to the factors in preference list of Marriage problem was solved in [8]. N-Factor Marriage problem with varying weight was introduced and solved in [9].

In this paper, a solution to a marriage problem where each member considers each factor for different proportion with Unequal Preference list has been discussed.

2. N-Factor Marriage Problem with Varying Weight and Unequal Preference List

A Marriage problem consists of two sets that is, men and women of size n. The n men consider a factor for more than one proportion in the preference list. Similarly n women consider a factor for more than one proportion in the preference list. This situation leads to form a matching problem with two or more preference lists. So, it becomes necessary to find out the correct matching between members of two groups considering the weightage of the factors in preference value matrix and also best possible solution for both men and women. The main objective of the study is to find out a satisfactory matching of a Marriage problem in which each member of men and women consider each factor (in N-factor) for different weightage.

To find out the correct optimum matching between men and women, considering all their preference factors, SMA with modification in step 6 is applied. The satisfactory value matrices with respect to each factor is added based on weightage of each factor and resultant satisfactory value matrix is obtained. The Hungarian algorithm is applied to this resultant matrix to get optimum satisfactory matching between men and women. An example of N-factor Marriage problem with varying weight is solved and discussed.

Example 1.1

Using modified SMA, determine satisfactory matching for an instance with four men m_1, m_2, m_3, m_4 and three women w_1, w_2, w_3 and three factors F_1, F_2, F_3 with the preference list. The members of men and women group give weight for each factor for different proportion as given below.

Factor-F_1				Factor-F_2				Factor-F_3			
m_1	50%	w_1	25%	m_1	30%	w_1	50%	m_1	20%	w_1	25%
m_2	40%	w_2	50%	m_2	25%	w_2	30%	m_2	35%	w_2	20%
m_3	30%	w_3	75%	m_3	35%	w_3	10%	m_3	35%	w_3	15%
m_4	20%			m_4	20%			m_4	60%		

Preference list based on Factor F_1

m_1 : w_1 w_2 w_3
m_2 : w_2 w_1 w_3
m_3 : w_3 w_2 w_1
m_4 : w_1 w_3 w_2

w_1 : m_4 m_1 m_2 m_3
w_2 : m_1 m_3 m_4 m_2
w_3 : m_3 m_2 m_1 m_4

Preference list based on Factor F_2

m_1 : w_1 w_3 w_2
m_2 : w_3 w_2 w_1
m_3 : w_2 w_3 w_1
m_4 : w_1 w_2 w_3

w_1 : m_1 m_4 m_2 m_3
w_2 : m_4 m_3 m_1 m_2
w_3 : m_3 m_4 m_1 m_2

Preference list based on Factor F_3

m_1 : w_3 w_1 w_2
m_2 : w_1 w_2 w_3
m_3 : w_2 w_3 w_1
m_4 : w_3 w_2 w_1

w_1 : m_2 m_3 m_4 m_1
w_2 : m_1 m_2 m_3 m_4
w_3 : m_3 m_4 m_1 m_2

Solution

The SMA is applied to the above instance for each factor of preference lists, preference value for each member is assigned based on their position in the preference list. The preference values are tabulated in the form of Men's Preference value matrix (PM_m) and Women's Preference value matrix (PM_w). The sum of PM_m and transpose of PM_w gives Satisfactory value matrix for each factor which are shown below.

The Satisfactory Value Matrix for Factor F_1 with weightage is

$$SM_{m(F_1)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{11}{16} & \frac{5}{6} & \frac{13}{24} \\ \frac{47}{120} & \frac{21}{40} & \frac{167}{240} \\ \frac{13}{80} & \frac{23}{40} & \frac{21}{20} \\ \frac{9}{20} & \frac{19}{60} & \frac{77}{240} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_2 with weightage is

$$SM_{m(F_2)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{4}{5} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{29}{120} & \frac{11}{40} \\ \frac{29}{120} & \frac{23}{40} & \frac{1}{3} \\ \frac{23}{40} & \frac{13}{30} & \frac{11}{40} \end{pmatrix} \end{array}$$

The Satisfactory Value Matrix for Factor F_3 with weightage is

$$SM_{m(F_3)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{47}{240} & \frac{4}{15} & \frac{11}{40} \\ \frac{3}{5} & \frac{23}{60} & \frac{37}{240} \\ \frac{73}{240} & \frac{9}{20} & \frac{23}{60} \\ \frac{13}{40} & \frac{9}{20} & \frac{57}{80} \end{pmatrix} \end{array}$$

In this instance, three factors are considered with different proportions by different members in preference list. So, the resultant satisfactory value matrix is formed by considering all weightage of factors and the satisfactory value matrix for each factor, which is given below.

The resultant Satisfactory Value Matrix is

$$SM_{m(R)} = \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} \frac{101}{60} & \frac{27}{20} & \frac{16}{15} \\ \frac{53}{40} & \frac{23}{20} & \frac{9}{0} \\ \frac{17}{24} & \frac{8}{5} & \frac{53}{30} \\ \frac{27}{20} & \frac{6}{5} & \frac{157}{120} \end{pmatrix} \end{array}$$

The Hungarian algorithm of assignment technique is applied to the above matrix and matchings are found. The satisfactory matching obtained on applying SMA is $(m_1, w_1), (m_3, w_3)$ and (m_4, w_2) and satisfactory value for each pair is given in the following table.

Satisfactory matching	Satisfactory value	Satisfactory value of men	Satisfactory value of women
(m_1, w_1)	$\frac{101}{60}$	$\frac{14}{15}$	$\frac{3}{4}$
(m_3, w_3)	$\frac{53}{30}$	$\frac{23}{30}$	1
(m_4, w_2)	$\frac{6}{5}$	$\frac{3}{5}$	$\frac{3}{5}$

Table 1.1: The preference value table of variants of Weighted N-factor Marriage problem

The above result shows that the satisfactory value of any member of matching in both groups on applying SMA algorithm is minimum of 3/5.The obtained solution is the optimum matching for both the groups and satisfactory level of men and women are 49.46% and 50.54% respectively.

3. CONCLUSION

In this study, N-Factor Marriage problem with varying weight and Unequal preference list has been introduced. SMA algorithm helps to find out the matching between men and women for a marriage considering many factors with varying weightage. Each factor was studied with a real life example. It was found that both men and women gain high satisfactory level and gets optimal matching. A Well known Assignment technique was used to solve the problems with N-Factors. This Technique helps the people to solve matching problems in real life situations and to take absolute decisions in a best possible manner.

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