MHD FLOW OF AN ELECTRICALLY CONDUCTING INCOMPRESSIBLE VISCOUS FLUID THROUGH POROUS MEDIUM BETWEEN TWO SEMI-INFINITE PARALLEL PLATES

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Abstract: The flow of an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium. Transverse magnetic field is applied perpendicular to the length of the plates. The flow of an electrically conducting incompressible viscous fluid under Transverse magnetic field produces induced electric current on which mechanical forces are exerted by the magnetic field. The induced current produces induce magnetic field and thus original magnetic field is also changes. Thus there is two way interactions between flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced current and the induced current changes the original magnetic field. . The expressions for velocity of the fluid, flow rate of the fluid, induced magnetic field and current density are obtained in elegant forms

Keywords: electrically conducting incompressible viscous fluid, porous medium, magnetic field, permeability parameter.

1. INTRODUCTION

The study of flow through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The classical Darcy's law Musakat [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P .$$

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiber glass, papus of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [2], Saffman [3] and others. A generalized Darcy's law proposed by Brinkmann [4] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{v} + \mu\nabla^2\vec{v}$$

Where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium.

The applications of flows through porous medium bears wide spread interest in Geophysics, biology and medicine. In many of these areas the flow consists of more than one phase, such type of flows find applications in the inter disciplinary fields such as bio-medical engineering etc., the flow of blood is one such application. The blood may be represented as Newtonian fluid and the flow of the blood is in two layered. Lightfoot [5], Shukla *et al.* [6] and Chaturani [7]. Bird *et al.* [8] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [9] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu*et al*. [10] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Transciet couette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwarbeg

et al. [11] Kandryzakaria*et al*. [12] discussed magneto hydrodynamics instability of interfacial waves between two immiscible cylindrical fluids.
Earlier Narasimhacharyulu*et al*. [13] studied the problem of two phase fluid flow between parallel plates with porous lining and Narasimhacharyulu*et al*. [14] examined the flow of micro polar fluid between parallel plates coated with porous lining.
In this present paper an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium. Transverse magnetic field is applied perpendicular to the length of the plates**.** The expressions for velocity of the fluid, flow rate of the fluid, induced magnetic field and current density are obtained in elegant forms

2. MATHEMATICAL FORMULATION OF THE PROBLEM

The flow of an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates .The space between the plates is filled with porous region. The coordinate system is taken such that x-axis lies parallel to the length of the plates and y-axis perpendicular to the length of the plates. The fluid flows under a constant pressure gradient.

$$G = -\frac{\partial p}{\partial x}$$

A transverse magnetic field is applied perpendicular to the flow of the fluid. The flow of an electrically conducting incompressible viscous fluid under Transverse magnetic field produces induced electric current on which mechanical forces are exerted by the magnetic field. The induced current produces induce magnetic field and thus original magnetic field is also changes. Thus there is two way interactions between flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced current and the induced current changes the original magnetic field

The velocity of the fluid $\vec{V} = (u,\ 0,\ 0)$ satisfies the equation of continuity, the physical quantities depend on y only.

The equation of motion is given by

$$\frac{d^2u}{dy^2} - \frac{u}{k} - \frac{\sigma\mu_e H_0}{\mu}u = -\frac{G}{\nu} \qquad (2.1)$$

$$-h < y < h$$

Induced magnetic field is given by the equation

$$\frac{1}{\mu}\frac{d^2h_x}{dy^2} + H_0\frac{du}{dy} = 0 \qquad \ldots(2.2)$$

Where $G = -\frac{\partial p}{\partial x}$ is a constant pressure gradient in the x direction, ν is coefficient of viscosity of the fluid, k is permeability of the porous medium h_x is induced magnetic field, μ_e magnetic permeability, σ electric conductivity, H_0 magnetic field strength component.

Using the following non-dimensional quantities.

$$u^* = \frac{uh}{\nu}, y^* = \frac{y}{h},\ G^* = \frac{Gh^3}{\nu},\ \beta^2 = \frac{h^2}{K},\ M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\mu} \quad \ldots \quad (2.3)$$

After removing *, the non-dimensional form of equation of motion is

$$\frac{d^2u}{dy^2} - \alpha^2 u = -\frac{G}{\nu}; \quad -1 < y < 1 \quad \ldots \quad (2.4)$$

$$where\ \alpha^2 = \beta^2 + M^2,\quad \beta^2 = \frac{h^2}{K},\quad M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\mu}$$

The boundary conditions are given by

$$\left.\begin{array}{lll} u = 0 & \text{at} & y = \pm 1 \\ h_x = 0 & \text{at} & y = \pm 1 \end{array}\right\} \quad \ldots \quad (2.5)$$

3. Solution of the problem

Solving the equations (2.2) (2.4)employing boundary conditions (2.5)

Velocity of the fluid is given by

$$u = \frac{G}{\nu\alpha^2}\left(1 - \frac{\cosh \alpha y}{\cosh \alpha}\right) \quad \ldots \quad (2.6)$$

Flow rate $Q = \int_{-1}^{1} u dy$

Flow rate of the fluid is given by

$$Q = \frac{2G}{\nu\alpha^2}\left(1 - \frac{Tanh \ \alpha}{\alpha}\right) \quad \ldots(2.7)$$

Where $\alpha^2 = \beta^2 + M^2, \quad \beta^2 = \frac{h^2}{K}, \quad M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\mu}$

Induced magnetic field is given by

$$\mu_x = \frac{\mu GH_0}{\nu\alpha^3}\left(\frac{\sinh(\alpha y)}{\cosh \alpha} - yTanh \ \alpha\right) \quad \ldots\ldots \quad (2.8)$$

Current density is given by

$$J = -\frac{dh_x}{dy} = \frac{\mu GH_0}{\nu\alpha^2}\left(\frac{Tanh \ (\alpha y)}{\alpha} - \frac{\cosh(\alpha y)}{\cosh \ \alpha}\right) \quad \ldots. (2.9)$$

CONCLUSION

Flow of an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates. The space between the parallel plates is filled with porous medium. Transverse magnetic field is applied perpendicular to the length of the plates.

From the obtained analytical solutions, it is observed that as the viscosity of the fluid is increasing the velocity of the fluid, flow rate and induced magnetic field are decreasing. Further it is also observed that as viscosity of the fluid is increasing, the current density is also decreasing

When fluid moves through a magnetic field, an electric field and consequently a current may be induced, and in turn the current interacts with the magnetic field to produce a body force on fluid. The production of this current has led to MHD power generators, MHD devices, nuclear engineering and the possibility of thermonuclear power has created a great practical need for understanding the dynamics of conducting fluids. The influence of a magnetic field in viscous incompressible flow of electrically conducting fluid is of use in extrusion of plastics in the manufacture of rayon, nylon etc

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