Two Phase Flow Of An Electrically Conducting Incompressible Viscous Fluid Between Two Semi-Infinite Parallel Plates Partially Filled With Porous Medium Under Transverse Magnetic Field

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Abstract: The aim of the present paper is to investigate flow of an electrically conducting incompressible viscous fluid between two semi-infinite parallel plates. The space between the parallel plates is partially filled with porous medium. The flow will be two phase flow one in clear region and other in porous region .Transverse magnetic field is applied perpendicular to the length of the plates. The flow of an electrically conducting incompressible viscous fluid under transverse magnetic field produces induced electric current on which mechanical forces are exerted by the magnetic field. The induced current produces induce magnetic field and thus original magnetic field is also changes. Thus there is two way interactions between flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced current and the induced current changes the original magnetic field. The expressions for velocities of the fluid in both regions, flow rate of the fluid, induced magnetic field in both regions and current density in both regions are obtained in elegant forms. The effect of magnetic parameter on fluid velocity is investigated. The results are graphically represented. It has been observed that viscosity of the fluid has significant role on the velocity profile**.**

Keywords: electrically conducting incompressible viscous fluid, porous medium, magnetic field, permeability parameter.

I. Introduction

The study of flows through porous medium assumed importance because of the interesting applications in the diverse fields of science, Engineering and Technology. The practical applications are in the percolation of water through soil, extraction and filtration of oils from wells, the drainage of water, irrigation and sanitary engineering and also in the inter disciplinary fields such as biomedical engineering etc. The classical Darcy's law Musakat [1] states that the pressure gradient pushes the fluid against the body forces exerted by the medium which can be expressed as

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla P$$

The law gives good results in the situations when the flow is uni-directional or the flow is at low speed. In general, the specific discharge in the medium need not be always low. As the specific discharge increases, the convective forces get developed and the internal stress generates in the fluid due to its viscous nature and produces distortions in the velocity field. In the case of highly porous medium such as fiber glass, papas of dandelion the flow occurs even in the absence of the pressure gradient.

Modifications for the classical Darcy's law were considered by the Beavers and Joseph [2], Saffman [3] and others. A generalized Darcy's law proposed by Brinkmann [4] is given by

$$O = -\nabla P - \left(\frac{\mu}{K}\right)\vec{v} + \mu\nabla^2\vec{v}$$

Where μ and K are coefficients of viscosity of the fluid and permeability of the porous medium

The applications of flows through porous medium bears wide spread interest in Geophysics, biology and medicine. In many of these areas the flow consists of more than one phase, such type of flows find applications in the inter disciplinary fields such as bio-medical engineering etc., the flow of blood is one such application. The blood may be represented as Newtonian fluid and the flow of the blood is in two layered. Lightfoot [5], Shukla *et al.* [6] and Chaturani [7]. Bird *et al*. [8] found an exact solution for the laminar flow of two immiscible fluids between two parallel plates. Bhattacharya [9] discussed the flow of immiscible fluids between rigid plates with a time dependent pressure gradient. Vajravelu *et al*. [10] have discussed the effect of magnetic field on unsteady flow of two immiscible conducting fluids between two permeable beds. Transient couette flow in a rotating non-Darcian porous medium parallel plate configuration is studied by Anwar beg *et al*. [11] Kandryzakaria *et al*. [12] discussed magneto hydrodynamics instability of interfacial waves between two immiscible cylindrical fluids.

Earlier Narasimhacharyulu *et al*. [13] studied the problem of two phase fluid flow between parallel plates with porous lining and Narasimhacharyulu *et al*. [14] examined the flow of micro polar fluid between parallel plates coated with porous lining.

In this present paper flow of an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates. The space between the parallel plates is partially filled with porous medium. The flow will be two phase flow one in the clear region and other in porous region. Transverse magnetic field is applied perpendicular to the length of the plates**.** The expressions for velocities of the fluid in both regions, flow rate of the fluid, induced magnetic field in both regions and current density in both regions are obtained in elegant forms

II.MATHEMATICAL FORMULATION OF THE PROBLEM

The flow of an electrically conducting incompressible viscous fluid is considered between two semi infinite parallel plates given by. y=$\pm h$ The space between the plates is filled partially with porous region. The coordinate system is taken such that x-axis lies parallel to the length of the plates and y-axis perpendicular to the length of the plates. The fluid flows under a constant pressure gradient.

A transverse magnetic field is applied in the porous region perpendicular to the flow of the fluid.

The velocity of the fluid $\vec{V} = (u, 0, 0)$ satisfies the equation of continuity, the physical quantities depend on y only.

The equation of motion in the two regions is given by

$$\frac{d^2up}{dy^2} - \frac{u_p}{k} - \frac{\sigma\mu_e H_0}{\mu} u_p = -\frac{G}{\nu} \quad \text{where} \quad -\delta < y < \delta \quad \ldots \quad (2.1)$$

$$\frac{d^2uc}{dy^2} - \frac{\sigma\mu_e H_0}{\mu} uc = -\frac{G}{\nu} \quad \text{where} \ -h < y < -\delta \ \text{ and } \ \delta < y < h \ \ldots \quad (2.2)$$

Induced magnetic field in the clear region given by the equation

$$\frac{1}{\mu}\frac{d^2h_x^c}{dy^2} + H_0\frac{du_c}{dy} = 0 \quad \text{where} \ -h < y < -\delta \ \text{ and } \ \delta < y < h \quad (2.3)$$

Induced magnetic field in the porous is given by the equation

$$\frac{1}{\mu}\frac{d^2h_x^p}{dy^2} + H_0\frac{du_p}{dy} = 0 \quad \text{where} \ -\delta < y < \delta \quad \ldots(2.4)$$

Where $G = -\frac{\partial p}{\partial x}$ is a constant pressure gradient, in the x direction, ν is coefficient of viscosity of the fluid, k is permeability of the porous medium. u_p and u_c are velocity of the fluid in the porous and clear region respectively , h_x^c is induced magnetic field in the clear region , h_x^p is induced magnetic field in the porous region , μ_e magnetic permeability, σ electric conductivity, and H_0 magnetic field strength component

using the following non-dimensional quantities.

$$u^* = \frac{uh}{\nu},\ y^* = \frac{y}{h},\ G^* = \frac{Gh^3}{\nu},\ \beta^2 = \frac{h^2}{K},\quad M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\mu} \quad \ldots \quad (2.5)$$

After removing *, the non-dimensional form of equation of motion is

$$\frac{d^2up}{dy^2} - \alpha^2 up = -\frac{G}{\nu}; \qquad -\frac{\delta}{h} < y < \frac{\delta}{h} \quad \ldots (2.6)$$

$$\frac{d^2uc}{dy^2} - M^2 uc = -\frac{G}{\nu}; \qquad -1 < y < -\frac{\delta}{h} \ \text{ and } \ \frac{\delta}{h} < h < 1 \quad \cdots \quad (2.7)$$

where $\alpha^2 = \beta^2 + M^2, \quad \beta^2 = \frac{h^2}{K}, \quad M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\mu}$

The boundary conditions are given by

$$\left.\begin{array}{ll} u_c = u_p \ \& \ h_x^p = 0 \quad \text{at} \quad y = \pm\dfrac{\delta}{h} \\ u_c = 0 \ \& \ h_x^c = 0 \quad \text{at} \quad y = \pm 1 \end{array}\right\} \quad \ldots \quad (2.8)$$

III.SOLUTION OF THE PROBLEM

Solving the equations (2.3)(2.4) and (2.6)(2.7) employing boundary conditions (2.8) we get

$$u_c = \frac{G}{\nu M^2}\left(1 - \frac{\cosh My}{\cosh M}\right) \quad \ldots \quad (2.9)$$

$$u_p = \frac{\cosh(\alpha y)}{\cosh(\alpha\delta/h)}\left\{\frac{G}{\nu M^2}\left(1 - \frac{\cosh(M\delta/h)}{\cosh M}\right) - \frac{G}{\nu\alpha^2}\right\} + \frac{G}{\nu\alpha^2} \quad \ldots \quad (2.10)$$

Flow rate $Q = \int_{-1}^{1} u\,dy$

$$Q = \int_{-1}^{-\delta/h} u_c dy + \int_{-\frac{\delta}{h}}^{0} u_p dy + \int_{0}^{\delta/h} u_p dy + \int_{\frac{\delta}{h}}^{1} u_c dy$$

$$Q = \frac{2G}{\nu M^2}\left(1 - \frac{\delta}{h} - MTan(hM) + M\frac{\sinh(M\,\delta/h)}{\cosh M}\right) + \frac{2}{\alpha}Tanh\left(\alpha\frac{\delta}{h}\right) \times$$
$$\times\left\{\frac{G}{\nu M^2}\left(1 - \frac{\cosh(M\delta/h)}{\cosh M}\right) - \frac{G}{\nu\alpha^2}\right\} + \frac{2G}{\nu\alpha^2}\left(\delta/h\right) \quad \ldots \quad (2.11)$$

Induced magnetic field in the clear region is given by

$$h_x^c = \frac{\mu G H_0}{\nu M^3}\left(\frac{\sinh(My)}{\cosh M} - yTanhM\right) \quad \ldots \ (2.12)$$

Current density in the clear region by

$$J^c = -\frac{dh_x^c}{dy} = \frac{\mu G H_0}{\nu M^2}\left(\frac{Tanh(My)}{M} - \frac{\cosh(My)}{\cosh M}\right) \quad \ldots. \ (2.13)$$

Induced magnetic field in the porous region is given by

$$h_x^p = \frac{\mu H_0 \alpha^3}{\cosh(\alpha\delta/h)}\left\{\frac{G}{\nu M^2}\left(1 - \frac{\cosh(M\delta/h)}{\cosh M}\right) - \frac{G}{\nu\alpha^2}\right\}\left\{\frac{\sinh(\alpha\,\delta/h}{\delta/h}y - \sinh(\alpha y)\right\} \text{--(2.14)}$$

Current density in the porous region by

$$J^p = -\frac{dh_x^p}{dy} =$$
$$\frac{\mu H_0 \alpha^3}{\cosh(\alpha\delta/h)}\left\{\frac{G}{\nu M^2}\left(1 - \frac{\cosh(M\delta/h)}{\cosh M}\right) - \frac{G}{\nu\alpha^2}\right\}\left\{\alpha\cosh(\alpha y) - \frac{\sinh(\alpha\,\delta/h}{\delta/h}y\right\}$$

…. (2.15)

IV.CONCLUSION

Flow of an electrically conducting incompressible viscous fluid is examined between two semi-infinite parallel plates. The space between the parallel plates is partially filled with porous medium. Transverse magnetic field is applied perpendicular to the length of the plates. The magneto hydrodynamic phenomena are a consequence of the mutual interaction of the fluid flow and the magnetic field. A conductor crossing magnetic field lines gives rise to an induced electric field, which drives an electric current in the conducting fluid. The resulting Lorentz force accelerates the fluid across the magnetic field, which in turn creates another induced electric field and currents which modify the initial magnetic field.

From the obtained analytical solutions, it is observed that as the viscosity of the fluid is increasing the velocity of the fluid in both regions, flow rate are decreasing. As viscosity of the fluid is increasing the induced magnetic field in both regions is decreasing Further it is also observed that as viscosity of the fluid is increasing, the current density in both regions is also decreasing. It has been observed that viscosity of the fluid has significant role on the velocity profile.

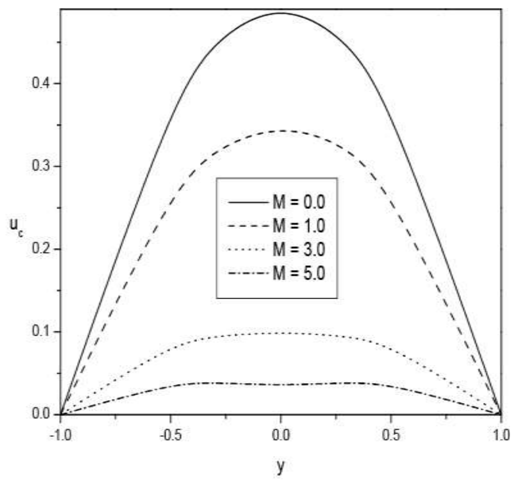

Fig. 1

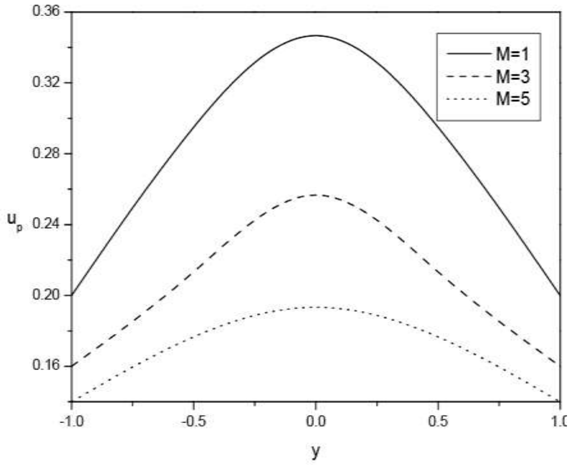

Fig. 2

Fig. 1 shows that velocity in clear medium is more parabolic when magnetic field is absent. The parabolic profile decrease with the increasing magnetic field.

Fig. 2 shows the velocity in the porous medium decreasing with increasing magnetic field values.

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