Topologies Generated by Triangular Neutrosophic Numbers

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Abstract. In this paper we have introduced a method to generate the topologies using the neutrosophic numbers,which is the extension work of the topologies generated by the intuitionistic fuzzy numbers. For this we have used (α, β, γ) -cut of neutrosophic number. Neutrosophic set plays a very important in recent days.The concept of neutrosophic set was putforth by F. Smarandache which has emerged as a new area of research in connection with topology.

Keywords: Neutrosophic Number; Triangular Neutrosophic Number; (α, β, γ)-cut of neutrosophic fuzzy Numbers

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1. Introduction

In reality, uncertainties and ambiguites exist everywhere. Zadeh's fuzzy logic Zadeh (1965) [10] has emerged as one of the important soft computing tools to describe the uncertainties.Fuzzy set uses only membership function.Several types of fuzzy numbers, e.g., triangular,trapezoidal,pentagonal,hexagonal, and octogonal, exist in the literature.In order to resolve the uncertainity about non-membership degrees, Atanassov (1986) [1] introduced intuitionistic fuzzy sets.Membership grades and non membership grades are discussed in maximum cases,but the indeterminacy degree has not been considered.And this is the base of considering neutrosophic set ,Neutrosophic set is the generalization of classical set, fuzzy set(1965), intuitionistic fuzzy set (1986) which consists of membership values, non-membership values, indeterminant values of a set. The concept of Neutrosophic set was first given by F. Smarandache (2005) [3]. Bera and Mahapatra introduced the (α, β, γ)- cuts of Neutrosophic sets. This paper is based on topologies generated by fuzzy numbers introduced by Padmapriya in 2015 [6] and Topologies generated by Intuitionistic fuzzy numbers by Santhi and Kungumaraj, 2020. [8]

2. Preliminaries

Definition 2.1. Neutrosophic set [3]
Let X be a nonempty set. A Neutrosophic set $\tilde{A}^N$ of X is defined as
$\tilde{A}^N = (x, \mu_{\tilde{A}^N}(x), \nu_{\tilde{A}^N}(x), \vartheta_{\tilde{A}^N}(x)); x \in X$ where $\mu_{\tilde{A}^N}(x)$, $\nu_{\tilde{A}^N}(x)$ and $\vartheta_{\tilde{A}^N}(x)$ are the membership function ,indetermianacy and non-membership function such that $\mu_{\tilde{A}^N}(x)$,$\nu_{\tilde{A}^N}(x), \vartheta_{\tilde{A}^I}(x) : X \to [0, 1]$ and $0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^N}(x) + \vartheta_{\tilde{A}^I}(x) \leq 3$ for all $x \in X$

Definition 2.2. Neutrosophic Number [3]

A Neutrosophic subset $\tilde{A}^N = (x, \mu_{\tilde{A}^N}(x), \nu_{\tilde{A}^N}(x), \vartheta_{\tilde{A}^N}(x)); x \in X$ of the real line R is called Neutrosophic number if the following conditions holds:

(i) There exist $x \in R$ such that $\mu_{\tilde{A}^N}(x) = 1$ and $\vartheta_{\tilde{A}^N}(x) = 0$

(ii) $\mu_{\tilde{A}^N}(x)$ is continuous function from $R \longrightarrow [0, 1]$ such that $0 \le \mu_{\tilde{A}^N}(x) + \nu_{\tilde{A}^N}(x) + \vartheta_{\tilde{A}^I}(x) \le 3$ for all $x \in X$

Definition 2.3. Triangular Neutrosophic Number

A Triangular Neutrosophic number $\tilde{A}^N$ is an Neutrosophic set in R with the following membership function $\mu_{\tilde{A}^N}(x)$, indetermiancy function$\nu_{\tilde{A}^N}(x)$ and non-membership function $\vartheta_{\tilde{A}^N}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, \; if \; a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, \; if \; a_2 \le x \le a_3 \\ 0, \; \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{"}}, \; if \; a_1^{'} \le x \le a_2 \\ \frac{x-a_2}{a_3^{"}-a_2}, \; if \; a_2 \le x \le a_3^{'} \\ 1, \; \text{otherwise} \end{cases}$$

$$\vartheta_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{'}}, \; if \; a_1^{'} \le x \le a_2 \\ \frac{x-a_2}{a_3^{'}-a_2}, \; if \; a_2 \le x \le a_3^{'} \\ 1, \; \text{otherwise} \end{cases}$$

where $a_1^{"} \le a_1{'} \le a_1 \le a_2 \le a_3 \le a_3^{'} \le_3$ " and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$, or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$, for all $x \in R$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3; a_1{'}, a_2, a_3{'})$.

Definition 2.4. α- Cut of a Neutrosophic Number

The α - cut of A denoted by ${}^{\alpha}A$ is the crisp set ${}^{\alpha}A = (x \in X : A(x) \ge \alpha)$ and the strong α - cut of A denoted by ${}^{\alpha+}A$ is the crisp set ${}^{\alpha+}A = (x \in X : A(x) > \alpha)$.

Definition 2.5. (α, β, γ)- cut of Neutrosophic Fuzzy Subset

A set of (α, β, γ)- cut generated by an Neutrosophic fuzzy subset A of X, where $(\alpha, \beta, \gamma) \in [0, 1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$A_{(\alpha,\beta),\gamma} = \{(x, \mu_{A^N}(x), \nu_{A^N}(x), \vartheta_{A^N}(x))\text{:}\; x \in X,\; \mu_{A^N}(x) \ge \alpha, \nu_{A^N}(x) \le \gamma,\; \vartheta_{A^I}(x) \le \beta, \alpha, \beta, \gamma \in [0, 1]\}$.

We define (α, β, γ) - level interval or (α, β, γ)- cut, denoted by $A_{(\alpha,\beta,\gamma)}$, as the crisp set of elements X which belong to A_N atleast to the degree α and which belong to A_N atmost to the degree β.

3. Topology generated by Neutrosophic Numbers

3.1. *Topologies generated by Neutrosophic Subsets*

Theorem 3.1. *If A is an Neutrosophic of X then the topology generated by the (α, β, γ) - cut of A is called the topology generated by the Neutrosophic subset A.*

Example 3.2. Let $X = (a, b, c)$ and $A = \{\frac{0.1}{a} + \frac{0.2}{b} + \frac{0.7}{c}; \frac{0.8}{a'} + \frac{0.7}{b'} + \frac{0.2}{c'}; \frac{0.1}{a"} + \frac{0.1}{b"} + \frac{0.1}{c"}\}$. Then ${}^{\alpha}A$ is the whole set X for $\alpha = 0$ and is the empty set for $\alpha = 1$. Also ${}^{\alpha}A = X$ for $0 < \alpha \leq 0.1$; ${}^{\alpha,\beta,\gamma}(A) = \{b, c\}$ for $0.1 < \alpha \leq 0.2$; ${}^{\alpha}A = \{c\}$ for $0.2 < \alpha \leq 0.7$;${}^{\alpha}A = \phi$ for $0.7 < \alpha < 1$. Then $\tau_I(A) = \{X, \{b, c\}, \{c\}, \phi\}$. Clearly $\tau_I(A)$ is topology on X.

Example 3.3. Let $X = (a, b, c)$ and $A = \{\frac{0.8}{q} + \frac{0.9}{r} + \frac{1}{s}; \frac{0.2}{q'} + \frac{0.1}{r'} + \frac{0}{s'}; \frac{0.2}{q'} + \frac{0.1}{r'} + \frac{0}{s'}\}$. Then $\tau_I(A) = \{X, \{r, s\}, \{s\}\}$ is not a topology on X.

3.2. **Topologies generated by the triangular Neutrosophic number**

Theorem 3.4. *If A is a triangular Neutrosophic number* $(a, c, b; a', c', b'; a", c", b")$ *then*
(i) $\tau_I(A) = \{R\} \cup \{[m, n] : [m, n] = min\{(x, y), (x', y'), (x", y")\}$ *and* $(x - a)(b - c) = (c - a)(b - y), (a' - x')(b' - c') = (c' - a')(b' - y'), (x" - a")(b" - c") = (c" - a")(b" - y"),$
$a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b', a" \leq x" \leq c", c" \leq y" \leq b"\}$.
(ii) $\tau_{I+}(A) = \{(a, b'), \phi\} \cup \{(m, n) : [m, n] = min\{(x, y), (x', y'), (x", y")\}$ *and* $(x - a)(b - c) = (c - a)(b - y), (a' - x')(b' - c') = (c' - a')(b' - y'), (x" - a")(b" - c") = (c" - a")(b" - y"),$
$a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b'\}$.

Proof *Consider the neutrosophic number* $(a, b, c; a', c', b')$. *The* (α, β) *- cut and strong* (α, β) *- cuts are given by* ${}^{0,0,1}A = R, A^{0,1,0} = (a', b, c'), A^{1,0,0} = (a, b'), A^{1,0,1} = (a, b', c), A^{1,1,0} = (a, b, c'), A^{0,1,1} = (a', b, c), A^{0+,0+,0+}A = a, b, c, A^{1+,1+}, A = \{\phi\}$. *Let* $A(x) = \frac{x-a}{c-a}; A(y) = \frac{b-y}{b-c}; A(x') = \frac{a'-x'}{c'-a'}; A(y') = \frac{b'-y'}{b'-c'}; A(x") = \frac{x"-a"}{c"-a"}; A(y") = \frac{b"-y"}{b"-c"}$.
Suppose $0 < \alpha, \beta, \gamma < 1$, *then* ${}^{(\alpha,\beta)}A = \{x : \mu_{A^I}(x) \geq \alpha, \vartheta_{A^I}(x) \leq \beta, \nu_{A^I}(x) \leq \gamma, \}$. *Choose x and y such that* $a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b', x" \leq a" \leq c", c" \leq y" \leq b"$ *with* $A(x) = A(y) = \alpha$, $A(x') = A(x') = \beta$ *and* $A(x") = A(x") = \gamma$.*Thus* ${}^{\alpha}A = [x, c] \cup [c, y]$, ${}^{\beta}A = [x', c'] \cup [c', y'],{}^{\gamma}A = [x", c"] \cup [c", y"]$.
Then ${}^{(\alpha,\beta,\gamma)}A = [m, n] = min\{(x, y), (x', y'), (x", y")\}$ *such that* $(x - a)(b - c) = (c - a)(b - y) = \alpha$, $(a' - x')(b' - c') = (c' - a')(b' - y') = \beta.(x" - a")(b" - c") = (c" - a")(b" - y") = \gamma$. *Thus* ${}^{(\alpha,\beta,\gamma)}A_N = [m, n]$ *and* ${}^{(\alpha,\beta,\gamma)}A_{N+} = (m, n)$ *such that* $a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b', a" \leq x" \leq c", c" \leq y" \leq b"$. *Then*
(i) $\tau_I(A) = \{R\} \cup \{[m, n] : [m, n] = min\{(x, y), (x', y'), (x", y")\}$ *and* $(x - a)(b - c) = (c - a)(b - y), (a' - x')(b' - c') = (c' - a')(b' - y'), (x" - a")(b" - c") = (c" - a")(b" - y")a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b', a" \leq x" \leq c", c" \leq y" \leq b"\}$.
(ii) $\tau_{N+}(A) = \{(a, b), \phi\} \cup \{(m, n) : [m, n] = min\{(x, y), (x', y'), (x", y")\}$ *and* $(x - a)(b - c) = (c - a)(b - y), (a' - x')(b' - c') = (c' - a')(b' - y'), (x" - a")(b" - c") = (c" - a")(b" - y")a \leq x \leq c, c \leq y \leq b, a' \leq x' \leq c', c' \leq y' \leq b', a" \leq x" \leq c", c" \leq y" \leq b"\}$.

Example 3.5. If A is a triangular neutrosophic number $[1, 2, 4; 1, 3, 5; 2, 4, 6]$ with

$$A_{\mu_{A^N}(x)} = \begin{cases} x-1, \ 1 \le x \le 2 \\ \frac{4-x}{2}, \ 2 \le x \le 4 \\ 0, \ otherwise \end{cases} \quad A_{\vartheta_{A^N}(x)} = \begin{cases} fracx-12, \ 1 \le x \le 2 \\ \frac{5-x}{2}, \ 2 \le x \le 4 \\ 0, \ otherwise \end{cases}$$

$$A_{\nu_{A^N}(x)} = \begin{cases} \frac{x-2}{2}, \ 2 \le x \le 4 \\ \frac{5-x}{2}, \ 4 \le x \le 6 \\ 0, \ otherwise \end{cases}$$

(i) $\tau_N(A) = R \cup \{[1.5, 3] : (x-a)(b-c) = (c-a)(b-y), 1 \le x \le 2$ and $2 \le y \le 4, 1 \le x^{'} \le 3$ and $3 \le y^{'} \le 5, 2 \le x^{"} \le 4and4 \le y^{"} \le 6\}$.

(ii) $\tau_{N+} = \{(1, 5), \phi\} \cup \{(1.5, 3) : (x-a)(b-c) = (c-a)(b-y), 1 \le x \le 2$ and $2 \le y \le 4, 1 \le x^{'} \le 3$ and $3 \le y^{'} \le 52 \le x^{"} \le 4and4 \le y^{"} \le 6\}$.

Corollary 3.6. *If A is a triangular Neutrosophic number $(a, c, b; a^{'}, c^{'}, b^{'}; a^{"}, c^{"}, b^{"})$ then $\tau_N(A)$ generates the topology on R and $\tau_{N+}(A)$ generates the topology on $(a, b^{'})$.*

Remark 3.7. If A is a triangular neutrosophic number $(a, c, b; a^{'}, c^{'}, b^{'}; a^{"}, c^{"}, b^{"})$ then it is evident that $\tau_N(A)$ and $\tau_{N+}(A)$ generates different topologies.

Proposition 3.8. *If A is a triangular neutrosophic number $(a, c, b; a^{'}, c, b^{'}; a^{"}, c, b^{"})$ and c is the midpoint of $[a, b]$, $[a^{'}, b^{'}]$ $[a^{"}, b^{"}]$ then*

(i) $\tau_N(A) = \{[c-\epsilon, c+\epsilon]; 0 \le 2\epsilon \le b-a\} \cup \{R\}$

(ii) $\tau_{N+}(A) = \{(c-\epsilon, c+\epsilon); 0 \le 2\epsilon \le b-a\} \cup \{\phi, (a, b)\}$

Proof:

Consider the neutrosophic number $(a, c, b; a^{'}, c, b^{'}; a^{"}, c, b^{"})$ where c is the midpoint of $[a, b]$, $[a^{'}, b^{'}]$ and $[a^{"}, b^{"}]$. The (α, β, γ) - cut and strong (α, β, γ) - cut are given by ${}^{0,0,1}A = R, {}^{0,1,0} = (a^{'}, b, c^{'}), A^{1,0,0} = (a, b^{'}), A^{1,0,1} = (a, b^{'}, c), A^{1,1,0} = (a, b, c^{'}), A^{0,1,1} = (a^{'}, b, c)A, {}^{0+,0+,0+}A = a, b, c, A^{1+,1+}A = \{\phi\}$. Let $A(x) = \frac{x-a}{c-a}; A(y) = \frac{b-y}{b-c}; A(x^{'}) = \frac{a^{'}-x^{'}}{c^{'}-a^{'}}; A(y^{'}) = \frac{b^{'}-y^{'}}{b^{'}-c^{'}}; A(x^{"}) = \frac{x^{"}-a^{"}}{c^{"}-a^{"}}; A(y^{"}) = \frac{b^{"}-y^{"}}{b^{"}-c^{"}}$.

since c is the midpoint of $[a, b]$, $[a^{'}, b^{'}]$ and $[a^{"}, b^{"}]$ m and n are at equal distance from c. So we can choose $\epsilon > 0$ such that $x = c-\epsilon, y = c+\epsilon, x^{'} = c-\epsilon$ and $y^{'} = c+\epsilon$,$x^{"} = c-\epsilon$ and $y^{"} = c+\epsilon$.

Therefore ${}^{(\alpha,\beta,\gamma)}A_N = [c-\epsilon, c+\epsilon]$ and ${}^{(\alpha,\beta,\gamma)}A_{N+} = (c-\epsilon, c+\epsilon)$; since $a \le m \le n \le b$ we have $m-n \le b-a$ that implies $2\epsilon \le b-a$. Therefore

(i) $\tau_N(A) = \{[c-\epsilon, c+\epsilon]; 0 \le 2\epsilon \le b-a\} \cup \{R\}$ and

(ii) $\tau_{N+}(A) = \{(c-\epsilon, c+\epsilon); 0 \le 2\epsilon \le b-a\} \cup \{\phi, (a, b)\}$

4. **Conclusion**

In this paper we have introduced an innovative technique to generate topologies using neutrosophic numbers. First we generated a topology using neutrosophic number then we have obtained the topology using triangular neutrosophic number. This paper provides the idea and act as bridge between topology and neutrosophic numbers. Further there is a scope to develop topological properties in terms of Neutrosophic numbers as well as triangular Neutrosophic numbers.

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