Strange quark matter cosmological model with constant deceleration parameter in *f(T)* theory of gravity.

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Abstract: In this study, we consider the Bianchi type-VI_0 space-time in the presence of strange quark matter with and without strings in the *f(T)* theory of gravitation. The two functional forms of the function *f(T),* which are the power model and linear model, are chosen for the investigation, and different forms of energy conditions are considered to obtain the exact solution of the nonlinear field equation. The various parameters i) average scale factor(t^{α}) ii) shear scalar(σ) and iii) expansion scalar(θ) with a constant deceleration parameter are studied. The dynamic behavior and geometrical features of the constructed model are discussed and presented using 3D graphs.

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1. Introduction

Cosmology and gravitation are two of the numerous scientific triumphs of the twentieth century. The present observational cosmological data [1],[2],[3],[4] supports the idea that the universe is accelerating. This accelerating expansion may be due to the presence of some secret energy, which is known as dark energy (DE). The effect of the DE can be determined in two ways. One is to modify Einstein's theory of gravitation, and the other is to build different De candidates. Here, the Einstein-Hilbert action has led us to various modified theories of gravitation, such as *f(R)*[5], [6], *f(T)*[7], *f(R,T)*[8] & *f(G)* gravity (where R is the scalar curvature, T is the torsion scalar, and G is the Gauss-Bonnet scalar).

Among these theories, the *f(T)* theory of gravity has received significant attention in recent years and is explained by the substitution of *f(T)* at the position of the torsion scalar T. The *f(T)* theory of gravity is a simple generalization of the TEGR, that is, the teleparallel equivalent of general relativity. A possible explanation for the accelerated expansion of the universe in *f(R)* gravity and the observed effects was discussed by S.Nojiri et al[9]. The dark side of gravity in *f(T)* gravity has been discussed by Francisco et al. [10]. R. Ferraro et al. [11] described the acceleration of the universe by considering the modified TEGR. M Sharif et al [12] observed the effect of an electromagnetic field on the Bianchi type-VI_0 universe. K. Bamba et al. [13] discussed a dark energy cosmological model using a cosmography test.

Recently, Cosmic String has developed an interest in the study of the behavior of the universe during its early evolution and late-time accelerated expansion of the universe [14], [15], [16], [17], [18], [19]. The cosmic string stored some amount of stress energy and then coupled it with the gravitational field, which developed interest in the study of the gravitational effect from the presence of a string. In addition, researchers have studied various string-coupled models in modified theories of gravity [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31]. Recently, some authors have performed excellent work on string cosmological models. Chirde *et al*. [32], [33] explained general relativity and quantum gravity using strange quark matter and string cosmological models in teleparallel gravity.

Transit dark-energy string cosmological models were discussed by Pradhan et al.. [34]. V.G.Mete et al suggested a string-cloud cosmological model for *f(T)* gravity[35]. Dabre *et al*. investigated the string model by considering various parameters[36], [37], [38] and observed that the universe may have undergone several phases of transition after the Big Bang explosion. These transitions produce a vacuum domain wall, string, and monopoles. Among these topological defects, strings play a vital role in the formation of galaxies. Therefore, their string models have received significant attention from researchers. Several authors have discussed the important aspects of string models in various theories of gravitation.

The origin of the cosmos is currently one of the most cosmological thrillers. What is the environmental position in the initial steps of our cosmos formation? This remains a question of study. According to measure-field theories, the damaged equilibrium can be recovered at a sufficiently high temperature. One event is the first-order phase transition from the quark phase to the hadron phase in the early universe, which forms a strange quark matter. Many researchers have investigated several ideas regarding this strange quark matter.

In modern physics, string theory is a theoretical explanation of point-like particles called strings. Strings have mass, charge, and other physical properties that depend on the vibrational state of strings that carry a certain gravitational force; thus, string theory is also known as the theory of quantum gravity. Therefore, in this study, we discuss the various types of strings and their properties.

In the following study, we attached a strange quark matter to the string cloud. It is important to connect the strange quark matter to the string cloud. The changes during the transition of the universe could be quark-gluon plasma (QGP) hadron gas (known as quark-hadron phase transition) when the cosmic heat was the $T \approx 200\ MeV$. In this model, we consider quarks as degenerate Fermi gas, which exists only in an arranged space equipped with a vacuum energy density B_c (Known as Bag constant). When we structured this model, the quark matter was composed of mass-less *u* and *d* quarks and massive *s* quarks, which are mass-less and non-interacting.

Therefore we have quark pressure

$$p_q = \frac{\rho_q}{3}\,, \tag{1}$$

Where, ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c\,. \tag{2}$$

But the total pressure is

$$p = \rho_q - B_c\,. \tag{3}$$

Dey *et.al.*[39] obtained *EoS* for a strange matter based on the model of the inter-quark potential. Mak and Harko [40] studied charged strange quark matter in spherically symmetric space-time admitting conformal motion. investigation strange quark matter packed into a string cloud in a spherical symmetry space-time accepting conformal move has been carried out by Yavuz *et al.* [41]. Adhav *et al*[42], [43] discussed string clouds and domain walls with quark matter in an N-dimensional Kaluza–Klein cosmological model and strange quark matter attached to string clouds in Bianchi type-III space-time in general relativity. Katore and Shaikh[44] and Rao and Neelima [45] obtained cosmological models with strange quark matter attached to cosmic strings for axially symmetric space-time. Khadekar and Rupali[46] discussed the geometry of quarks and strange quark matter for higher-dimensional space-time in general relativity. Recently, Pawar *et al.* [47] discussed a string cosmological model with a zero-mass scalar field under *f(R)* gravity.

2. Methodology and Equation of Motion

In this section, we provide the field equation for *f(T)* theory of gravity obtained from the action given by [48].

$$I = \int e[f(T) + L_m]d^4x \,, \tag{4}$$

where *f(T)* is a differentiable function of the torsion scalar T,L_m is Lagrangian matter, and $e = \sqrt{-g}$.

The line element for a general space-time is defined as

$$ds^2 = g_{ij}dx^i dx^j \,, \tag{5}$$

where g_{ij} is a component of the symmetric metric tensor. The above line element can be transformed into Minkowski space-time as

$$ds^2 = g_{ij}dx^i dx^j = \eta_{ij}\theta^i\theta^j \,, \tag{6}$$

$$ds^2 = e_k^i\theta^k \,, \theta^k = e_i^k dx^i \,, \tag{7}$$

η_{ij} is a Minkowskian space-time such that $\eta_{ij} = diag[1, -1, -1, -1]$ and

$$e_k^i e_j^k = \delta_j^i \,. \tag{8}$$

$\sqrt{-g} = \det[e_i^k] = e$ and the dynamic field of the tetrad matrix e_i^m. The Weitzenbocks connection components, which have zero curvature but nonzero torsion for a manifold, are defined as

$$\Gamma^m{}_{ij} = e_k^m \partial_j e_i^k = -e_i^k \partial_j e_k^m \,. \tag{9}$$

Here, the torsion and the antisymmetric tensors are, respectively, defined as

$$T^m{}_{ij} = \Gamma^m{}_{ij} - \Gamma^m{}_{ij}$$

$$= e_k^m(\partial_i e_j^k - \partial_j e_i^k) \tag{10}$$

$$S_m{}^{ij} = \frac{1}{2}\left(K^{ij}{}_m + \delta_m^i T^{nj}{}_n - \delta_m^j T^{ni}{}_n\right) . \tag{11}$$

The contorsion tensor tensor is defined as

$$K^{ij}{}_m = -\frac{1}{2}\left(T^{ij}{}_m - T^{ji}{}_m - T_m{}^{ij}\right) . \tag{12}$$

The torsion scalar is defined using the contraction, which is similar to the scalar curvature in GR as

$$T = T^m{}_{ij} S_m{}^{ij} \,. \tag{13}$$

Equations of motion are obtained by functional variation of the action concerning the tetrads as

$$S_i{}^{jp}(\partial_p T)f_{TT} + \left\{e^{-i}e_i^q \partial_p\left(e e_k^m S_m{}^{jp}\right) + T^m{}_{\lambda i} S_m{}^{j\lambda}\right\} f_T + \frac{1}{4}\delta_i^j f = 4\pi T_i^j \tag{14}$$

Where T_i^j is the energy-momentum tensor for a strange quark matter attached to a string cloud [41], f_T and f_{TT} are the first and second derivatives of $f(T)$ with respect to T . If we take *f(T)* = constant, then the equation of motion (14) is reduced to the equation of motion of teleparallel gravity with a cosmological constant, which is dynamically equivalent to general relativity.

3. Metric and field equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI_0 line element

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2e^{2x}dz^2. \tag{15}$$

Where *(x,y,z,t)* are Cartesian coordinates, and *A,B &C* are functions of cosmic time "t" only.

The energy-momentum tensor for a string cloud [50] is given by

$$T_{ij} = \rho u_i u^j - \rho_s x_i x_j \tag{16}$$

Here, ρ is the rest energy density, and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s \tag{17}$$

Where ρ_p is the particle energy density.

When we study various vibration models of a string representing different types of atoms, these models can be seen as different masses or spins. Thus, in this case, quarks are used instead of atoms in the string cloud. As a result, in the string cloud, we consider the quark matter energy density rather than the atom energy density.

In this case, from (17), we get

$$\rho = \rho_q + \rho_s + B_c, \tag{18}$$

Where ρ_q is the quark energy density and B_c is the bag constant, which takes a value between 60 and 80 $\frac{MeV}{(fm)^3}$.

From (17) and (18), we obtain the energy-momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_s + B_c)u_i u_j - \rho_s x_i x_j \tag{19}$$

Where x_i is a unit space-like vector that describes the direction of the string.

We have u_i and $x_i$with satisfying conditions:

$$g^{ij}u_i u_j = -x^i x_j = 1 \text{ and } u^i x_j = 0 \tag{20}$$

Here, we admit the direction of the string along the x-axis. So, we have

$$f + 2f_T\left[\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - 2\right] + 2\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]\dot{T}f_{TT} = 0\,.$$

The torsion scalar from (13) is obtained as

$$T = -2\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - 1\right] \tag{21}$$

Using co-moving coordinates, the field equation (14) for metric (15) generates the following series of equations:

$$f + 2f_T\left[\frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - 1\right] + 2\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right]\dot{T}f_{TT} = 16\pi\rho_s\,. \tag{22}$$

$$f + 2f_T\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + 2\frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} - 1\right] + 2\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right]\dot{T}f_{TT} = 0\,. \tag{23}$$

$$f + 2f_T\left[2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + 1\right] + 2\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right]\dot{T}f_{TT} = 0\,. \tag{24}$$

$$f + 4f_T\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right] = 16\pi\rho\,. \tag{25}$$

$$\frac{1}{2}\left[\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right]f_T = 0\,. \tag{26}$$

The overhead dot represents Ordinary differentiation with respect to cosmic time *'t'*.

4. Solutions of field equation

Strange quark cosmological model of Bianchi type-*VI₀* with string cloud

From Eq.(26) one can get

$$B = k_1 C \tag{27}$$

where k_1 is a constant of integration. The constant k_1, without loss of generality, can be chosen as unity [50] so thus, we have:

$$B = C \tag{28}$$

From Eqs. (21) – (27), we obtain the following equations:

$$f + 4f_T\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 - 1\right] + 4\left[\frac{\dot{B}}{B}\right]\dot{T}f_{TT} = 16\pi\rho_s\,. \tag{29}$$

$$f + 4f_T\left[2\frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2\right] = 16\pi\rho\,. \tag{30}$$

$$f_T\left[\frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} + 1\right] - \left[\frac{\dot{B}}{B}\right]\dot{T}f_{TT} = 4\pi\rho_p\,. \tag{31}$$

From eq. (30)–(32), we obtain three nonlinear differential equations with six unknowns, namely A, B, *f*, ρ_s, ρ, ρ_p. Therefore, additional conditions are required to solve these equations.

The shear scalar σ^2 is proportional to the scalar expansion θ so that we can take the relationship between the metric potentials *A* & *B* Collins *et al.*[51]

$$A = B^n \tag{32}$$

Where $n \neq 0$ is a constant.

We consider the form of the average scale factor as

$$a(t) = t^\alpha \tag{33}$$

Where α is the positive constant.

The spatial volume is defined as,

$$V = ABC = AB^2 = A^{\frac{n+2}{n}} \tag{34}$$

The relationship between the average scale factor $a(t)$ and spatial volume *(V)* is defined as

$$a(t) = V^{\frac{1}{3}} \tag{35}$$

$$a(t) = A^{\frac{n+2}{3n}} = t^{\alpha} \tag{36}$$

$$A = (t^{\alpha})^{\frac{3n}{n+2}} \tag{37}$$

$$B = C = (t^{\alpha})^{\frac{3}{n+2}} \tag{38}$$

Initially, the Scale factor $a(t)$ is zero at $t = 0$. Hence, it has a large band-type singularity.

5. Model I: $f(T) = \eta(-T)^{\beta}$

Where β is a positive constant.

This model is reliable compared with the model of ΛCDM model given by Zhang er al.[52]

From eq. (21), we have obtained the torsion scalar as

$$T = -2\left[\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right] \tag{39}$$

$$f(T) = \eta\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta} \tag{40}$$

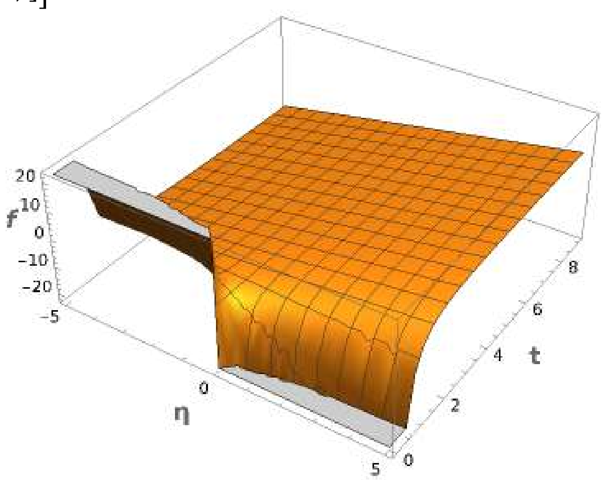

Fig.1 Plot of *f(T)* Vs *t* (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

The graph of *f(T)* presented by the 3D graph shown in fig.1 tends toward a constant value over a large time limit. Thus, the 3D graph shows that the surface of *f(T)* is compressed with respect to the small value of the torsion scalar.

From eq.(29)-(31), string density is given as

$$\rho_s = \frac{\eta}{16\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{\frac{\beta(\beta-1)432\alpha^3(2n+1)}{(n+2)^2t^4}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{-2} + 4\beta\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{-1}\left[\frac{3\alpha(3\alpha-1)}{(n+2)t^2} - 1\right] + 1\right\} \tag{41}$$

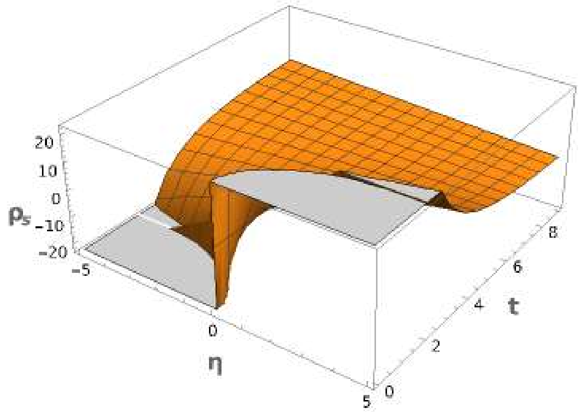

Fig.2 Plot of ρ_s(kg m^{-1}s^{-1}) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig.(2) shows that the behavior of the string density ρ_s is increasing for $\eta < 0$ and decreases for $\eta > 0$ with respect to time *t*.

The string energy density is given as

$$\rho = \frac{\eta}{16\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} \tag{42}$$

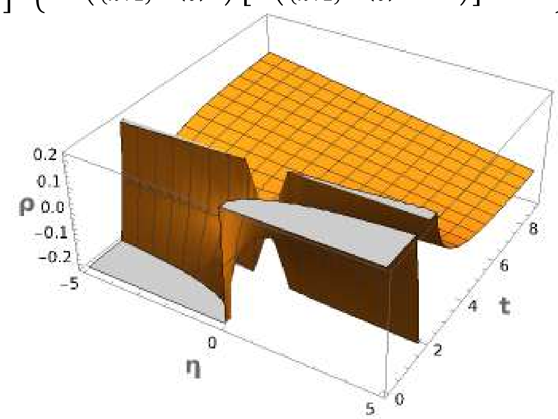

Fig.3 Plot of ρ (kg m^{-1}s^{-1}) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig. (3) indicates that the energy density ρ is an increasing function for $\eta < 0$ and a decreasing function for $\eta > 0$ with respect to time *t*.

The particle energy density is given as

$$\rho_p = \frac{\beta\eta}{4\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta-1}\left\{\left[\frac{9(n-1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 + \frac{3\alpha}{(n+2)t^2} + 1\right] - (\beta-1)\frac{108\alpha^2}{(n+2)^3 t^4}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{-2}\right\} \tag{43}$$

Quark energy density is

$$\rho_q = \rho - B_c$$

$$= \frac{\eta}{16\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} - B_c \tag{44}$$

Quark Pressure is

$$p_q = \frac{\rho_q}{3}$$

$$p_q = \frac{\eta}{48\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} - \frac{B_c}{3} \tag{45}$$

Total pressure is

$$p = p_q - B_c$$

$$= \frac{\eta}{48\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]^{-1}+1\right\}-\frac{4B_c}{3} \quad (46)$$

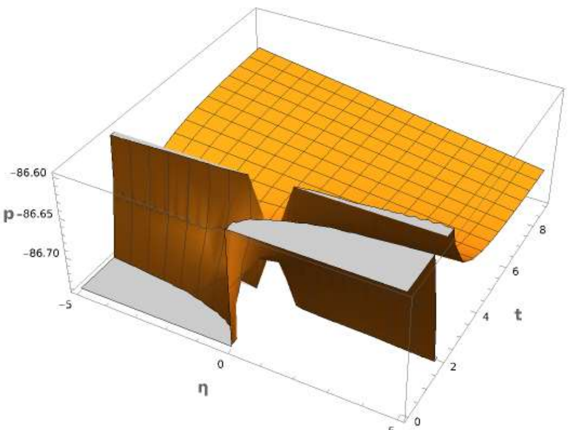

Fig.4 Plot of p (kg/cm^2) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig.(4) shows that the pressure p is negative for all values of η.

Fig. (3) and (4) show that the energy density ρ tends to zero, and the pressure p is negative throughout the evolution of the universe, representing the expansion of the universe.

6. Model II: $f(T) = e^{mT}$

Where m is the arbitrary constant and T is the Torsion Scalar

This model was studied by Zhang et al.[52] who observed that the ΛCDM model does not follow a model that is considered as a standard model of the expanding universe.

$$f(T) = e^{mT}$$

$$= e^{m\left(-2\left(\frac{9(2n+1)}{(n+2)^2}\right)\left(\frac{\alpha}{t}\right)^2-1\right)}$$

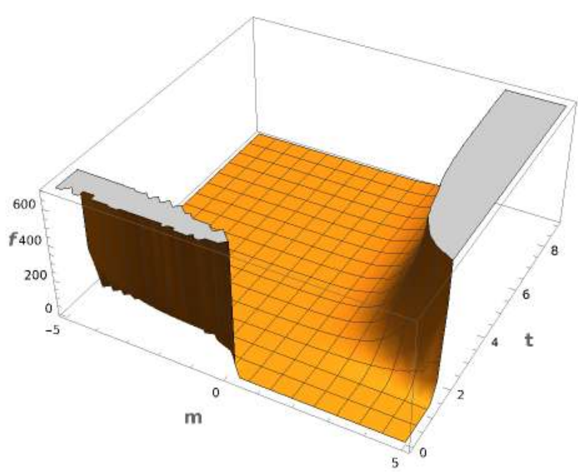

Fig.5 Plot of *f(T)* Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig. (5) shows that *f(T)* tends to a constant value of zero for a long time for all values of η which indicates that the surface of *f(T)* compresses with respect to the values of the torsion scalar.
From eq.(30)-(32), string density is given as

$$\rho_s = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{16}.\left[1+\frac{432m^2\alpha^3(2n+1)}{(n+2)^3t^4}+4m\left(\frac{3\alpha(3\alpha-1)}{(n+2)t^2}-1\right)\right] \quad (47)$$

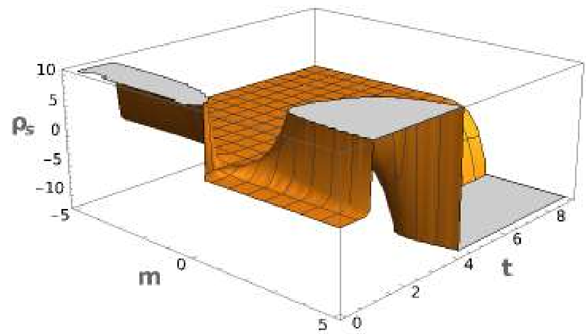

Fig.6 Plot of ρ_s(kg m^{-1}s^{-1}) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig.(6) indicates that string density ρ_s is a decreasing function for all values of η with respect to time.

The string energy density is given as

$$\rho = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{16}.\left[1+36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2t^2}\right)\right] \quad (48)$$

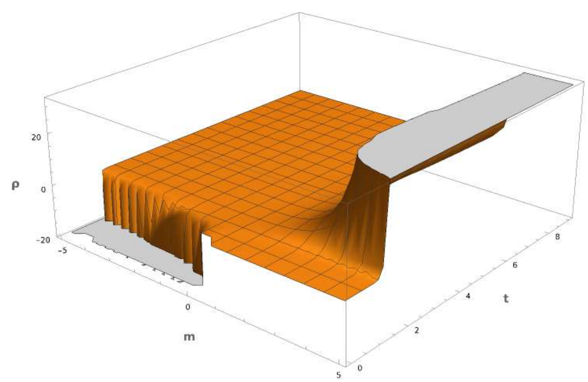

Fig.7 Plot of ρ(kg m^{-1}s^{-1}) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig.(7) shows that the energy density ρ initially approaches a large value and then tends to a constant value over a large time and is positive throughout the evolution of the universe for all values of η.

The particle energy density is

$$\rho_p = \frac{me^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{4\pi}.\left[1+\frac{9(n-1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2+\frac{3\alpha}{(n+2)t^2}+m\frac{108\alpha^3(2n+1)}{(n+2)^3t^4}\right] \quad (49)$$

Quark energy density is

$$\rho_q = \rho - B_c$$

$$\rho_q = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{16\pi}.\left[1+36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2t^2}\right)\right]-B_c \quad (50)$$

Quark Pressure is

$$p_q = \frac{\rho_q}{3}$$

$$p_q = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{48\pi}.\left[1+36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2t^2}\right)\right] - \frac{B_c}{3} \quad (51)$$

Total pressure p is

$$p = p_q - B_c$$

$$p = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2-1\right)\right]}}{48\pi}.\left[1+36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2t^2}\right)\right] - \frac{4B_c}{3} \quad (52)$$

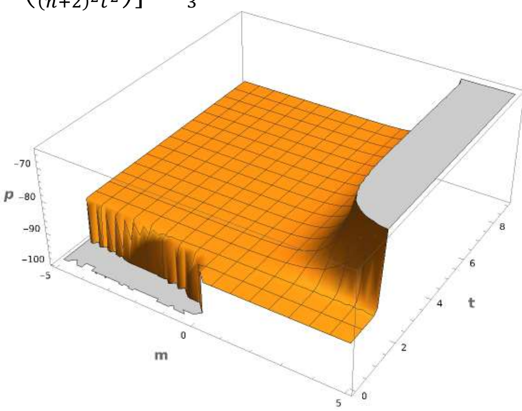

Fig.8 Plot of p(kg/cm^2) Vs t (Gyr) for $\alpha = 1.2, \beta = 1$ & $n = 1$.

Fig. (8) Describes that the pressure p is a negative & increasing function with respect to time t for all values of η.

From figs. (7) and (8), the pressure p and energy density ρ are positive. This indicates that pressure and energy density would produce an attractive gravitational effect, while negative pressure represents the expansion of the universe in this model.

7. The Energy Conditions

The energy conditions are mathematically set limits, which claim that energy is positive. There are various cases of energy conditions associated with pressure and density. In this section, we discuss the energy conditions defined by Cappizzielleo *et al.*[53] defines.

The energy conditions are given by

i) Null Energy Condition (NEC) $\rho + p \geq 0$
ii) Weak Energy Condition (WEC) $\rho \geq 0$ & $\rho + p \geq 0$
iii) Strong Energy Condition (SEC) $\rho + 3p \geq 0$ & $\rho + p \geq 0$
iv) Dominant Energy Condition (DEC) $\rho \geq 0$ & $\rho \pm p \geq 0$.

For Model I: $f(T) = \eta(-T)^{\beta}$

From eq.s (42)-(46) we express the energy conditions for linear model as

$$\rho + p = \frac{\eta}{12\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} - \frac{4B_c}{3}. \quad (53)$$

$$\rho + 3p = \frac{\eta}{8\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} - 4B_c. \quad (54)$$

$$\rho - p = \frac{\eta}{24\pi}\left[2\left[\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]\right]^{\beta}\left\{4\beta\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2\right)\left[2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]^{-1} + 1\right\} + \frac{4B_c}{3}. \quad (55)$$

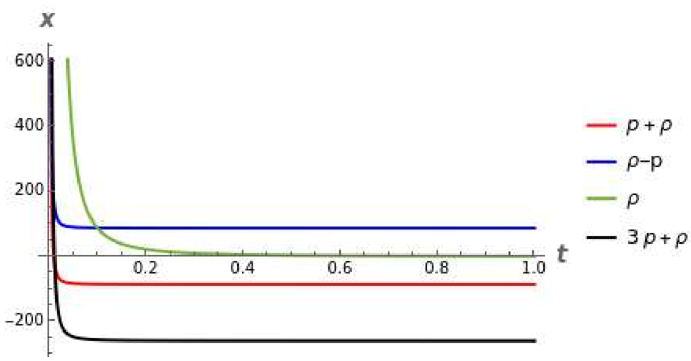

Fig.9 Plot of $\rho + p$, ρ-p, ρ, $\rho + 3p$ Vs t

All the fig. are for fixed constant $\alpha = 1.2, \beta = 1, n = 1, \& \eta = 5$**.** The graphical behavior of the energy conditions is shown in fig.9.

From the figures, it is clear that NEC, WEC, and SEC were completely violated, and in this case, the energy density became positive throughout the expansion of the universe. From. fig.9 $\rho - p \geq 0$ & $\rho \geq 0$ which satisfied DEC. Naturally, the presence of the DEC and violation of the SEC show that the universe is dominated by unknown energy and matter, which is confirmed by fig.9 and the viability of describing the universe's accelerated expansion. A similar study was conducted by other authors[54], [55].

Model II: $f(T) = e^{mT}$

From eq.s (48)-(52) we express the energy conditions for linear model as

$$\rho + p = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]}}{12\pi}.\left[1 + 36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2 t^2}\right)\right] - \frac{4B_c}{3}. \quad (56)$$

$$\rho + 3p = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]}}{8\pi}.\left[1 + 36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2 t^2}\right)\right] - 4B_c. \quad (57)$$

$$\rho - p = \frac{e^{m\left[-2\left(\frac{9(2n+1)}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 - 1\right)\right]}}{24\pi}.\left[1 + 36m\left(\frac{\alpha^3(2n+1)}{(n+2)^2 t^2}\right)\right] + \frac{4B_c}{3}. \quad (58)$$

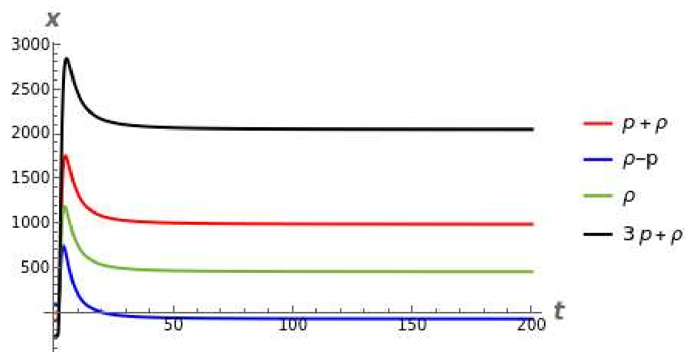

Fig.10 Plot of $\rho + p$, ρ-p, ρ, $\rho + 3p$ Vs t

All the fig. are for fixed constant $\alpha = 1.2, \beta = 1, n = 1, \& m = 5$. The graphical behavior of the energy conditions is shown in fig.10.

From the figure, it is clear that NEC, WEC, SEC, and DEC were completely satisfied. Figure 10 shows that the SEC is satisfied. The above graphs show that WEC and NEC are satisfied with our derived model and are physically acceptable. In addition, fig.10 shows the condition of the DEC that was initially fulfilled. All these energy conditions are positive because of the consideration of the exponential model and are satisfied because of the behavior of the dark energy in our universe for comic acceleration. Similar results have been obtained by various authors[56], [57].

From Fig. 9 and 10, for both models, the effective energy densities ρ are positive during the entire epoch with respect to the cosmic time *(t),* which represents an accelerated expansion of the universe.

8. Some important properties of both models.

The matric (15) can be written as

$$ds^2 = dt^2 - (t^\alpha)^{\frac{6n}{n+2}} dx^2 - (t^\alpha)^{\frac{6}{n+2}} e^{-2x} dy^2 - (t^\alpha)^{\frac{6}{n+2}} e^{2x} dz^2 \quad (59)$$

Equation (15) represents a four-dimensional Bianchi type-VI$_0$ model with strange quark matter coupled with a string cloud in the *f(T)* theory of gravity. In the discussion of cosmology, we discuss the following physical and geometrical parameters. Our model is helpful in discussing the evolution of the universe.

8.1 Spatial volume *(V)* for our model is given by

$$V = ABC = (t^{\alpha})^3 \tag{60}$$

The volume of the model increases rapidly for any positive constant α which represents the volume of the expanding universe, as shown in fig.11.

8.2 Average scale factor *(a(t))* is

$$a(t) = V^{\frac{1}{3}} = t^{\alpha} \tag{61}$$

8.3 Hubble parameter *(H)* is

$$H = \frac{H_1+H_2+H_3}{3} = \frac{\alpha}{t} \tag{62}$$

where H_1, H_2 & H_3 are the directional Hubble parameters that express the expansion rates of the universe in the x, y, and z directions, respectively.

8.4 Anisotropic parameter *(A_h)*

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 = \frac{2}{3}\frac{(2+(n+1)^2)}{(n+2)^2} \tag{63}$$

If $A_h = 0$, the universe expands isotropically. However, here, we considered $n > 0$; which gives a non-vanishing anisotropic parameter and shows that our model is anisotropic throughout the expansion of the universe.

8.5 Expansion Scalar (θ)

$$\Theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = 3\left(\frac{\alpha}{t}\right) \tag{64}$$

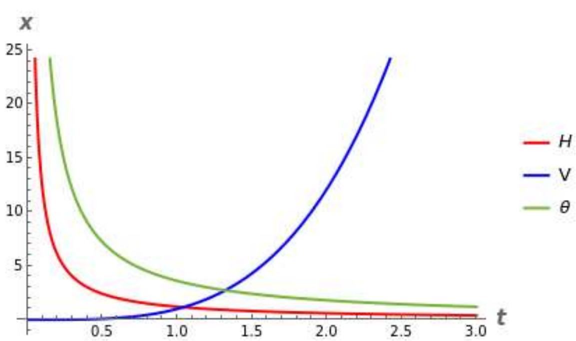

Fig.11 Plot of *H*, Θ and *V* Vs. *t* (Gyr) for $\alpha = 1.2$

8.6 Shear Scalar (σ) is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - \frac{\theta^2}{3}\right) = \frac{3}{2}\frac{(3n^2+5)^2}{(n+2)^2}\left(\frac{\alpha}{t}\right)^2 \tag{59}$$

8.7 The deceleration parameter *(q)* is

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{1}{3\alpha} - 1 \quad (60)$$

The deceleration parameter of our observed model was negative, indicating that the universe was accelerating. Subsequently, it tends to zero, indicating a constant rate of expansion. Recent observational data are consistent with our observations [58]. Recently SNe Ia observation shoes that present universe is accelerating and the value of DP lies in the range $-1 \leq q < 0$[59].

Concluding Remarks:

In this study, we considered the *f(T)* gravity field equations for Bianchi type-VI_0 in the presence of strange quark matter, with and without strings. The solution to the field equation was obtained by considering two different models: linear and exponential.

- For model I, the energy density (ρ) is positive, and the pressure (p) is negative, which indicates the accelerated expansion of the universe.
- For Model II, both the energy density (ρ) and pressure (p) are negative, which indicates that the pressure and energy density would produce an attractive gravitational effect on the expansion of the universe.
- For both models, the energy density(ρ) and string density(ρ_s) tend to be constant values to the late time, which indicates that the string phase disappears in the late time, as supported by [60]. Later, string density(ρ_s) approaches zero, which is evidence that our built universe is dominated by unknown matter, namely dark matter [61], [62]. This is in good agreement with recent observational data.
- Initially the model has Big Bang type singularity.
- For Model I, NEC, WEC, SEC, and DEC were completely violated and DEC was satisfied.
- For Model II, the NEC, WEC, SEC, and DEC were completely satisfied.
- For both models I and II, the spatial volume *(V)* is zero at $t = 0$ and then increases with respect to cosmic time *t,* which indicates that the universe is expanding, supporting recent observations [63], [64], [65], [66], [67].
- The plot of Hubble's parameter (*H*) and the expansion scalar (Θ) diverge at an early time and then reach a constant value at a later time *t* as shown in fig.17. The Hubble parameter tends to a constant value as $t \to \infty$, such that the universe asymptotically approaches the De-Sitter space. $\lim_{t\to\infty} \frac{\sigma}{\theta} = 0$, this condition was formulated by Collins *et al.*[68] for homogeneity and isotropy, which is satisfied in the present model.
- From eq.(63), the average anisotropic parameter (A_h) for both models is constant and does not vanish, indicating that both models are anisotropic throughout the evolution of the universe [69], [70].
- The deceleration parameter is constant for both models, which confirms recent results [71], [72], [73], which show that the expansion of the universe will be continuous forever.

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Data availability statement .

All data generated or analyzed during this study are included in this published article in the introduction as well as in the in-text citation [supplementary file is in the reference section].

Ethics Statement

This research is purely theoretical in nature and focuses exclusively on the domains of cosmology and general relativity. It does not involve any human or animal subjects, clinical trials, or practical applications requiring ethical approval. Consequently, no ethical concerns arise from the methodology, and institutional ethical clearance was not necessary.

Conflict of interest: The authors declare that they have no Conflict of interest.

Consent to Publish: Not applicable

"This research is purely theoretical and does not involve any practical applications or human subjects; hence, clinical trials are not required."

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