A STUDY OF SEIRS EPIDEMIC MODEL WITH HOLLING -TYPE III FOR COVID -19

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Abstract: In case of an outbreak of COVID -19 Pandemic, Psychological or Inhibitory effects and vaccination play a major role in controlling the impact of the COVID -19 on society. We developed a Susceptible – Exposed – Infected-Recovered–Susceptible type epidemic model with explicitly vaccination control. The transmission of infectious disease from susceptible to infected is taken Holling-type III Incidence rate. The Holling- type III Incidence rate is to interpret the psychological or inhibitory effect on the population. In their steady-state the model has two equilibrium point namely Disease-Free Equilibrium and Endemic equilibrium. Detailed analysis of the epidemic model is performed using the Basic reproduction number, centre manifold theory and Routh - Hurwitz criterion. When Basic Reproduction Number is less than unity then disease will be dies-out. The disease will persist when the Basic Reproduction number is greater than unity. At R_0=1, transcritical bifurcation is occur and Geometric approach is used to obtain the global stability condition of the system**.** Sensitivity analysis of R_0 is analysed and numerical simulation is also done.

Key words: Epidemic Model, Equilibrium, Lyapunov function, centre-manifold theorem, Reproduction number.

1. INTRODUCTION

From the beginning of the COVID-19 (SARS-COV-2), Infectious disease has a huge effect on the lifestyle of human beings. The novel corona virus disease (COVID-19) was first confirmed in the China City of Wuhan, late December 2019. The rapidity of its spread in many countries around the globe made the WHO declare it as a Global Pandemic with the invention of different control tools like vaccination, treatments, quarantine, lockdown etc. through the severity of the corona virus infectious disease reduces but the total control is yet to be achieved. The corona virus disease (COVID-19) caused by the severe acute respiratory syndrome corona virus-2 (SARS-COV-2) present clinical features which are similar to the disease caused by other corona viruses severe acute respiratory syndrome (SARS) and middle east respiratory syndrome such as lower respiratory illness with fever, dry cough, shortness of breath etc.

Many Mathematical models have appeared in an attempt to assess the dynamic mathematics models aimed at estimating of the basic reproduction number [17,10,8,1]. Many researchers work on vaccination control and how much they are effective.

In this paper we aim to study the dynamical behaviour of a corona virus diseases which has some effective vaccine and can recovered either by naturally or by some proper treatments. Corona virus disease is that whenever the virus enters into the body of a susceptible person will be exposed for some time and leave in latent period then become infectious some rate. The total human population can be divided into four time dependent individual classes susceptible S(t), exposed E(t), infected class I(t) and recovered class R(t). The incidence in an epidemic model is the rate at which the susceptible become infectious and it play an important role in transmission of disease. We consider transmission of infectious disease from susceptible to infected is taken Holling type-III Incidence rate in the form

$$\frac{\beta SI}{1+\eta I^2}, (\beta, \eta > 0)$$

Where η is the half saturation constant and β is known as the transmission parameter. βSI Measure the force of infection and $\frac{1}{1+\eta I^2}$is measure the inhibition effect of the behavioural change of the susceptible individuals where there is an increase in the no. of infective. Let A be the recruitment rate at any time *t* and μ be the available vaccination control which are applied to the newly recruited person. The portion μA, are vaccinated and become immune from the disease (This immune may be temporary though) and move to the recovered class. But we assume that the recovery is not permanent and the vaccination may be temporary immunity. The portion (1–μ)A remain susceptible for the disease. Some other rates are described in Table 1.

Table: 1 Description of parameter

Parameter	Description of parameter
A	The recruitment rate
μ	The available vaccination control which are applied to the newly recruited person
σ	rate at which exposed become infected
D	Natural death rate
β	the transmission parameter
η	is the half saturation constant

γ	disease related death rate is γ
α	Rate at which recovered loss immunity and go to the susceptible class
M	rate at which infected individuals recover either by naturally or by treatment

In the mathematical model transmission of disease is represented by flow diagram in figure 1.

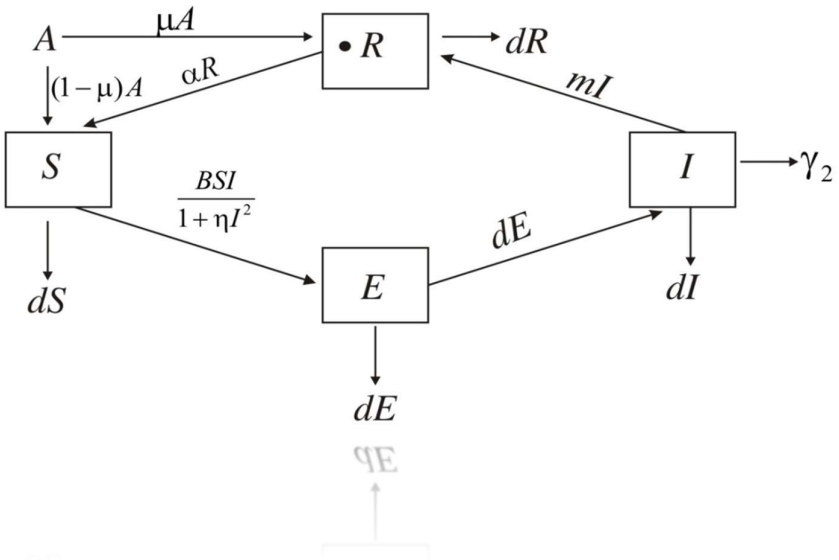

Figure: 1 Flow diagram of disease transmission

The Mathematical Model for corona virus infection disease is described by the following system of differential equations:

$$\frac{ds}{dt} = (1-\mu)A - ds - \frac{\beta SI}{1+\eta I^2} + \alpha R$$

$$\frac{dE}{dt} = -dE + \frac{\beta SI}{1+\eta I^2} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - dI - \gamma I - mI$$

$$\frac{dR}{dt} = A\mu - dR + mI - \alpha R \qquad \text{... (1)}$$

2. POSITIVITY AND BOUNDEDNESS OF THE SYSTEM

Positivity:

Lemma 2.1: Solution of the model equations (1) together with initial conditions of the system are:

$$S(0) > 0, E(0) \geq 0, I \geq 0, R \geq 0 \quad \text{... (2)}$$

Are always positive that is the model is positive for all *t*.

Boundedness:

Theorem 2.2: The solution of the system (1) is uniformly bounded.

Proof: Suppose $X = S + E + I + R$

$$\frac{dX}{dt} = A - dX$$

The feasible region of system (1) is

$$\psi = \left\{(S + E + I + R): \lim_{\eta \to \infty} \sup(S + E + I + R) \leq \frac{A}{d}\right\} \quad \text{... (3)}$$

Thus is naturally follows that the region is ψ positively invariant with respect to system (1). Positivity and Boundedness of solution of a system (1) well behaved.

2.3 Basic Reproduction Number

Basic reproduction number is defined as the average number of secondary infections caused by a single infectious individual during their entire infectious period and it is denoted by R_0. Basic reproduction number measure of how quickly an epidemic will take off. We calculated the basic reproduction number of the system (1) by Next Generation matrix [18].

The system has a always a disease-free equilibrium $E_0(S_0, 0, 0, R_0)$

Where, $S_0 = \dfrac{A(d(1-\mu)+\alpha)}{d(d+\alpha)}$ and $R_0 = \dfrac{A\mu}{d+\alpha}$

To calculate basic reproduction number we write down the system (1) as follows:

$$\frac{dx}{dt} = F(x) - V(x)$$

Where $F(x) = (E, I, R, S)T$

$$F(x) = \begin{bmatrix} \frac{BSI}{1+\eta I^2} \\ 0 \end{bmatrix}, V(x) = \begin{bmatrix} (\sigma + d)E \\ -\sigma E + (d + m + \gamma)I \end{bmatrix}$$

Jacobian matrix of $F(x)$ & $V(x)$ at E_0 are respectively

$$f(x)_{E_0} = \begin{bmatrix} 0 & \dfrac{\beta A(d(1-\mu)+\alpha)}{d(d+\alpha)} \\ 0 & 0 \end{bmatrix}$$

$$V(x)_{E_0} = \begin{bmatrix} (d+\sigma) & 0 \\ -\sigma & (d+m+\gamma) \end{bmatrix}$$

The basic reproduction number of the system (i) is defined by the spectral radius of the matrix

$$R_0 = \rho(F.V^{-1}) = \frac{\sigma\beta A(d(1-\mu)+\alpha}{d(d+\alpha)(\sigma+d)(d+m+\gamma)} \quad \text{..... (4)}$$

3. EQUILIBRIUM OF THE MODEL AND THEIR STABILITY

The system (1) has two possible non-negative equilibrium. The first one is the disease free equilibrium $E_0(S_0, 0,0, R_0)$ and other one is Endemic equilibrium

$$E_*(S^*, E^*, I^*, R^*)$$

Where, $S* = \frac{(\sigma+d)(\gamma+m+d)}{\sigma\beta}(1+\eta I^{*2})$

$$E^* = \frac{(\gamma+m+d)}{\sigma} I^*$$

$$I^* = \frac{-[\beta(d+\alpha)(d+\sigma)(d+\gamma+m)-\beta\sigma\alpha m]+\sqrt{\Delta}}{2\eta d(d+\sigma)(d+\alpha)(d+\gamma+m)}$$

Where, $\Delta = [\beta(d+\alpha)(d+\sigma)(d+\gamma+m)-\beta-\alpha m]^2$

$$-4d^2\eta(d+\alpha)^2(d+\sigma)^2(d+m+\gamma)^2(1-R_0)$$

$$R^* = \frac{A\mu+mI^*}{(d+\alpha)}$$

Clearly, I^* will have positive roots if only if

$\Delta > 0$ *and* $R_0 > 1$. Then Endemic equilibrium $E*$ is exists if $R_0 > 1$.

Theorem 3.1: If R_0 is less than unity then disease free equilibrium is locally asymptotically stable and unstable if $R_0 > 1$.

Proof: The characteristics equation to the system (1) at E_0 is given by

$$(d+\lambda)(d+\alpha+\lambda)[\lambda^2+(d+\alpha)(d+\gamma+m)+\lambda(2d+\gamma+m+\sigma)$$

$$-\frac{\sigma\beta A(d(1-\mu)+\alpha)}{d(d+\alpha)}] = 0$$

$$(d+\lambda)(d+\alpha+\lambda)[\lambda^2+(2d+\gamma+m+\sigma)\lambda+(d+\sigma)(d+\gamma+m)(1-R_0)] = 0.$$

... (5)

From above characteristics equation two roots are negative real part

$\lambda = -d, and\ \lambda = -(d+\alpha)$ And two other roots are depends on

$\lambda^2 + P\lambda + Q = 0$, where P=$(2d + \gamma + m + \sigma)$ and

Q=$(d + \sigma)(d + \gamma + m)(1 - R_0)$

If R0 <1, then both roots are negative real part and system (1) is locally asymptotically stable.

If R0>1 then one has positive real part and other one is negative real part and system (1) goes unstable.

Theorem 3.2: If R_0 is less than unity then disease free equilibrium is Globally Asymptotically Stable.

Proof: Construct a Lyapunov function

$$L = \sigma E + (\sigma + d)I$$

$$L' = \sigma E' + (\sigma + d)I'$$

$$= \left(\frac{\beta SI}{1 + \eta I^2} - (\sigma + d)E \right)\sigma + (\sigma + d)\left(\sigma E - (d + \gamma + m)I\right)$$

$$= I * \left(\frac{R_0}{1 + \eta I^2} - 1 \right)(d + \gamma + m)(d + \sigma)$$

If $R_0 < 1$ then $L' < 0$ for all point in ψ other than E_0.

Therefore, from LaSalle's invariance principle disease-free-equilibrium is Globally Asymptotically Stable.

3.3 Analysis at $R_0 = 1$.

In this section is to analyse the behaviour of the system (1) when R_0 =1. The Jacobian matrix of the system (1) evaluated at R_0 =1 and parameter

$$\beta = \beta^* = \frac{d(d+\sigma)(d+\alpha)(d+\gamma+m)}{\sigma A(\alpha+(1-\mu)d)}$$

has one of the Eigen value as zero and the remaining Eigen values are negative stability at point R_0 = 1 applying centre manifold theory (Sastry 1999).

We consider system (1)

$S = x_1, E = x_2, I = x_3$ And $R = x_4$ the *n*, the system (1) can be rewritten as follows.

$$\frac{dx_1}{dt} = (f - \mu)A - dx_1 - \frac{Bx_1x_3}{1 + \eta x_3^2} + \alpha x_4$$

$$\frac{dx_2}{dt} = -dx_2 + \frac{Bx_1x_3}{1 + \eta x_3^2} - \sigma x_2$$

$$\frac{dx_3}{dt} = -dx_3 + \sigma x_2 - \gamma x_3 - mx_3$$

$$\frac{dxy}{dt} = A\mu - dx_4 + mx_3 - \alpha x_4 \quad \text{... (7)}$$

Let J* be the Jacobian matrix evaluated at $R_0 = 1$ and $\beta = \beta^*$. On addition let $\mu = [\mu_1, \mu_2, \mu_3, \mu_4]$ and $w = [w_1, w_2, w_3, w_4]^T$ be the left Eigen vector and right Eigen vector of J* corresponding to zero Eigen value

$$\begin{bmatrix} -d & 0 & -\frac{\beta A(d(1-\mu)+\alpha}{d(d+\alpha)} & \alpha \\ 0 & -(d+\alpha) & \frac{\beta A(d(1-\mu)+\alpha)}{d(d+\alpha)} & 0 \\ 0 & 6 & -(d+\gamma+m) & 0 \\ 0 & 0 & m & -(d+2) \end{bmatrix}$$

Then, we have following:

$$\mu_1 = 0, \mu_2 = 1, \mu_3 = 0, \mu_4 = 0$$

$$w_1 = -\left[\frac{(\sigma+d)^2(d+\gamma+m)+\sigma m\alpha}{d(d+\gamma+m)(d+\alpha)}\right]$$

$$w_2 = 1, w_3 = \frac{\sigma}{d+\gamma+m}, w_4 = \frac{m\sigma}{(d+\gamma+m)(d+\alpha)}$$

The non-zero, partial derivatives associated with the function of the system (J) and from Castillo Chavez and song (2004), the bifurcation constant a_1 and b_1 are as follows.

$$a_1 = \sum_{K,i,j=1}^{\mu} \mu_k w_i w_j \left(\frac{\partial^2 f_k}{\partial x_i \partial x_j}\right)_{E_0}$$

$$= \frac{-2\sigma\beta^b}{d+\gamma+m}\left[\frac{(d+\sigma)^2(d+\gamma+m)}{d(d+\gamma+m)(d+\alpha)} + \sigma m\alpha\right] < 0$$

$$b_1 = \sum_{K,i=1}^{\mu} \mu_k w_i \left(\frac{\partial^2 f_k}{\partial x_i \partial \beta^*}\right)_{E_0}$$

$$= \frac{A(d(1-\mu)+\alpha)}{d(d+\alpha)} > 0$$

Then the disease free equilibrium changes its stability from stable to unstable at $R_0 = 1$ and there exists a positive equilibrium as R_0 cross one. Hence the system (7) exhibits transcritical bifurcation with bifurcation parameter β^* at $R_0 = 1$

Theorem 3.3: If R_0 is greater than unity, then the Endemic equilibrium $E_*(S^*, E^*, I^*, R^*)$ is locally asymptotically stable.

Proof: Jacobian of system (J) at the K endemic state E and E_* is

$$J(E^*) = \begin{Bmatrix} -d - \frac{\beta I}{1+\eta I^2} & 0 & -\frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} & \alpha \\ \frac{\beta I}{1+\eta I^2} & -(\sigma + d) & \frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} & 0 \\ 0 & \sigma & -(\gamma + m + d) & 0 \\ 0 & 0 & m & -(d+\alpha) \end{Bmatrix}$$

From which we obtain the characteristics equation

$$\lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D = 0 \quad \text{... (8)}$$

$$A_1 = \left(4d + \alpha + \sigma + m + \gamma + \frac{\beta I^*}{1+\eta I^2}\right)$$

$$B = (\sigma + d)(\gamma + m + d) + \left(d + \frac{\beta I}{1+\eta I^2}\right)(d+\alpha) +$$

$$\left(2d + \alpha + \frac{\beta I^*}{1+\eta I^2}\right)(2d + \sigma + m + \gamma) - \frac{\sigma \beta^* S(1-\eta I^2)}{(1+\eta I^2)^2}$$

$$C = \left(d + \frac{BI^*}{1+\eta I^2}\right)(d+\alpha)(2d+\sigma+m+\gamma) + \left(2d + \alpha + \frac{\beta I^*}{1+\eta I^2}\right)$$

$$(\sigma + d)(\gamma + m + d) + \sigma\left(\frac{\beta I^*}{1+\eta I^2}\right)\left(\frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2}\right)$$

$$-\sigma \frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2}\left(2d + \alpha + \frac{\beta I^*}{1+\eta I^*}\right)$$

$$D = \left(d + \frac{\beta I^\eta}{d+\eta I^\eta}\right)(d+\alpha)(d+\sigma)(\gamma+m+d)$$

$$-\frac{\sigma\beta S(1-\eta I^2)}{(1+\eta I^2)^2}\left(\frac{d+\beta I^*}{d+\eta I^2}\right)(d+\alpha)+\sigma\left(\frac{BI'}{1+\eta I^2}\right)x$$

$$\frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2}(d+\alpha)-m\alpha\sigma\frac{\beta I}{1+\eta I^2}$$

Thus, by direct computation, we have that $ABC > C^2 + A^2D.$ Therefore, by Routh-Hurwitz criterion, if follows that the endemic equilibrium E^*, of (1) is locally asymptotically stable.

GLOBAL STABILITY OF ENDEMIC EQUILIBRIUM

Definition 3.1: The system (1) is said to be uniformly persistent in ψ, if there exists a constant $c > 0$ such that any solution $(S(t), E(t), I(t), R(t))$of system (1) with initial

Value $\big(S(0), E(0), I(0), R(0)\big) \in \text{int}\big(S(0), E(0), I(0), Q(0), R(0)\big) \in \text{int}\,\psi$ satisfies.

$$\min\left\{\liminf_{t\to\infty} S(t), \liminf_{t\to\infty} E(t), \liminf_{t\to\infty} I(t), \liminf_{t\to\infty} R(t)\right\} \geq c.$$

Similar to [7], we can get

Theorem 3.4: System (1) is uniformly persistent in ψ if and only if $R_0 > 1$.

Remarks 3.1. The uniform persistence of system (1) in the bounded set Ω is equivalent to the existence of a compact $K \subset \psi$ that is absorbing for (1) (see [10]). Next we investigate the global stability of the endemic equilibrium of model (1). A geometrical approach developed in [13] (see also [12, 14, 6]) for proving global stability will be used in our discussion. Such method is based on the use of a high-order generalization of the Bendixons criterion which precludes the existence of non-constant periodic solutions (see [19]).

Now we briefly outline a general mathematical framework developed in [15] for proving global stability. Consider the autonomous dynamical system.

$$\frac{dx}{dt} = f(x)$$

Where $x \to f(x) \in R^n$ is a C^1 function about x in $\psi_1 \subset R^n$. Assume that the following hypothesis is holds:

(H_1): There is a compact absorb set $K \subset \psi_1$;

(H_2): Differential equation (*) has a unique equilibrium x_* in ψ_1.

Let $x \to P(x)\binom{n}{2} \times \binom{n}{2}$ matrix-valued function that is C^1 for$x \in \psi_1$. Assume that $P^{-1}(x)$ exists and is continuous for $x \in K$. A quantity q is defined as

$$q = \lim_{t\to\infty} \sup \sup_{x\in K} \frac{1}{t} \int_0^1 \mu(B(x(s,x_0)))ds$$

Where

$B = P_f P^{-1} + P\frac{\partial f[2]}{ax} p^{-1}$, The matrix P_f is obtained by replacing each entry P_{ij} of P by its derivative in the direction of f. The quantity $\mu(B)$ is the *Lazinski* measure of B with respect to a vector norm $|\cdot|$ in $R^N, N = \binom{n}{2}$, defined by $\mu(B) = \lim_{t\to 0+} \frac{|I + hB| - 1}{h}$.

The following global stable result is proved in Theorem 3.5 or [13].

Lemma 3.4: Suppose that ψ_1 is simply connected and that assumption (H_1)–(H_2) hold, then the unique equilibrium x_* is globally stable ψ_1 in if $q < 0$.

Now we apply the theory developed in [15], in particular Lemma 3.4, to prove the global stability of E^*.

Theorem 3.5: If R_0 greater than unity, then Endemic equilibrium E* is globally asymptotically stable.

Proof: We consider Sub-system of (1)

$$\frac{dS}{dt} = (1-\mu)A - dS - \frac{\beta SI}{1+\eta I^2} + \alpha R$$

$$\frac{dE}{dt} = \frac{BSI}{1+\eta I^2} - (d+\sigma)E$$

$$\frac{dI}{dt} = \sigma t - (\gamma + m + d)I \qquad \ldots (9)$$

The Jacobian of system (9) is

$$J(E_*) = \begin{bmatrix} -\left(d + \frac{\beta I}{1+\eta I^2}\right) & 0 & -\frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} \\ \frac{\beta I}{1+\eta I^2} & -(\sigma+\alpha) & \frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} \\ 0 & \sigma & -(\gamma+m+d) \end{bmatrix} \qquad \ldots (10)$$

And its second additive matrix is

$$J^{[2]} = \begin{bmatrix} -\frac{\beta I}{1+\eta I^2} - m & \frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} & \frac{\beta S(1-\eta I^2)}{(1+\eta I^2)^2} \\ \sigma & \frac{-\beta I}{1+\eta I^2} - \eta & 0 \\ 0 & \frac{\beta I}{1+\eta I^2} & -K \end{bmatrix}$$

Where, $m = 2d + \sigma$

$n = 2d + m + \gamma$

$K = 2d + m + \gamma + \sigma$

Choose the function $P(x) = P(S,E,I) = diag\left(1, \frac{E}{I}, \frac{E}{I}\right)$

$$P_f P^{-1} = diag\left(0, \frac{E^1}{E} - \frac{I^1}{I}, \frac{E^1}{E} - \frac{I^1}{I}\right)$$

$$PJ^{[2]}, P^{-1} \Rightarrow \begin{bmatrix} -\frac{\beta I}{1+\eta I^2} - m & \frac{BSI(1-\eta I^2)}{E(1+\eta I^2)^2} & \frac{\beta SI(1-\eta I^2)}{E(1+\eta I^2)^2} \\ \sigma\frac{E}{I} & -\frac{\beta I}{1+\eta I^2} - \eta & 0 \\ 0 & \frac{\beta I}{1+\alpha I} & 1-K \end{bmatrix}$$

$B = P_F P^{-1} + PJ^{[2]}P^{-1}$ Can be written in matrix from

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Let (μ, v, w) be vector in R* its norm $|\,|-|\,|$ is defined as

$$\|\mu, v, w\| = \max\{|\mu|, |v| + |w|\}$$

Let $\mu(B)$ be the Lozinski measure with respect to this norm then as described

$$\mu(B) \le \sup\{g_1, g_2\}$$

We received this $\frac{1}{t}\int_0^t \mu(B)ds \le \frac{1}{t}\int_0^t \left(\frac{E^1}{E} - d\right)ds$

This implies $q \le -\frac{\mu}{2} < 0$

We know that equilibrium E_* is globally asymptotically stable.

$$\frac{dR}{dt} = A\mu - dR + mI - \alpha R \quad \text{... (11)}$$

and its limit system is based on (11)

We get $$R(t) = \frac{A\mu + mI}{d + \alpha}$$

Then we get that E* is globally asymptotical stable. We proof the global stability of endemic equilibrium of model (1) with geometric approach [16]. Then it conclude our system is stable.

4. Sensitivity Analysis and Numerical simulation

Sensitivity indices allow us to measure the relative change in a state variable when a parameter changes. The normalized forward sensitivity index of a variable to a parameter is the ratio of the relative change in the variable to the relative change in the parameter. When the variable is a differentiable function of the parameter, the sensitivity index may be alternatively defined using partial derivatives.

4.1 Definition: The normalized forward sensitivity index of a variable, h, that depends differentially on a parameter, l, is denoted by $\mathbf{\Gamma}_l^h$ defined as:

$\mathbf{\Gamma}_l^h = \frac{l}{h} \times \frac{\partial h}{\partial l}$

Sensitivity indices of R_0

We drive the sensitivity of R_0 to each of the different parameter $\mu, \alpha, \gamma, m, \beta, \sigma$.

The sensitivity index of R_0 with respect to μ is presented by

$\mathbf{\Gamma}_\mu^{R_0} = \frac{\mu}{R_0} \times \frac{\partial R_0}{\partial \mu} = \frac{-d\mu}{\alpha + (1-\mu)d}$

Sensitivity indices of R_0 to other parameters of the model are shown below:

$\mathbf{\Gamma}_\alpha^{R_0} = \frac{\alpha}{R_0} \times \frac{\partial R_0}{\partial \alpha} = \frac{\alpha d\mu}{(d+\alpha)(a+(1-\mu)d)}$ And

$$\mathbf{\Gamma}_m^{R_0} = \frac{-m}{m+\gamma+d}, \quad \mathbf{\Gamma}_\gamma^{R_0} = \frac{-\gamma}{m+\gamma+d}, \mathbf{\Gamma}_\mu^{R_0} = \frac{-d\mu}{a+(1-\mu)d}, \mathbf{\Gamma}_\sigma^{R_0} = \frac{d}{d+\sigma}$$

$\mathbf{\Gamma}_\beta^{R_0} = 1$

The parameter is arranged from most sensitive to least sensitive. From the sensitivity indices most sensitive parameters are β and σ which is the maximum contact rate between susceptible and infected class and transfer rate from exposed to infected class respectively. The least sensitive parameter is disease related death rate γ. The parameter that change R_0 can be consider as control parameters. Table 2 describe the sensitivity indices of parameter to R_0.

Table: 2 Description of sensitivity indices

Parameter	***Description***	***Sensitivity indices***	***Increasing/decreasing in R_0***
β	Contact rate between susceptible and infected	1	Increment(decrement) in contact rate gives increment (decrement) in R_0
σ	Transfer rate from exposed to infected class	0.11	Increment(decrement) in transfer rate gives increment(decrement) in R_0
α	Rate at which recovered loss immunity and go to the susceptible class	0.088	Increment (decrement) in recovered loss immunity rate gives increment (decrement) in R_0
μ	The available vaccination control which are applied to the newly recruited person	-0.11	Increment(decrement) in vaccination control gives decrement(increment) in R_0
M	Rate which infected population recover naturally	-0.941	Increment(decrement) in recovery rate gives decrement(increment) in R_0
γ	Disease related death rate	-0.011	Increment (decrement) in disease related death decrement (increment) R_0

Now the analytical result is verified with the help of numerical simulation. We Consider a parameter set as Q_1=$\{\mu$,A,d,$\eta, \gamma, \alpha, \sigma, m\}$ ={0.5,100,0.1,0.9,0.025,0.4,0.8,2}. Some of the parameter is taken from the previous published paper and some are assumed. For $\beta = 0.001$ we have only one disease free equilibrium E_0 (900, 10, 10, 20) and it is locally asymptotically stable in figure 2. Again taking same parameter set Q_1, with $\beta = 0.01$ we have two feasible region one is disease free and another is endemic equilibrium is locally stable in figure 3.

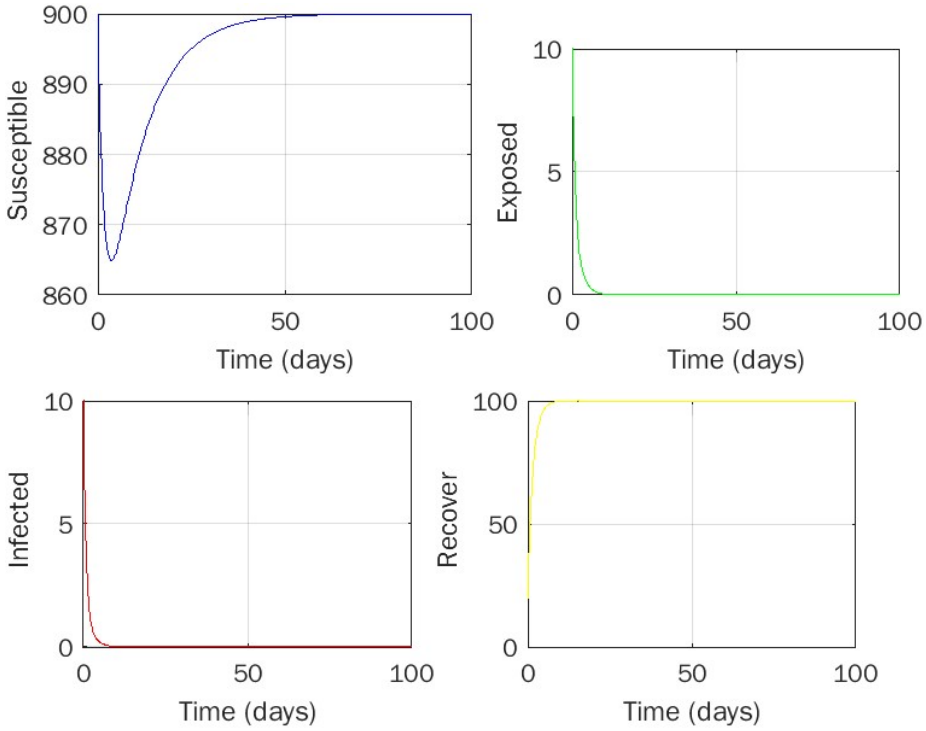

Figure 2: Disease free equilibrium with respect to parameter Q_1.

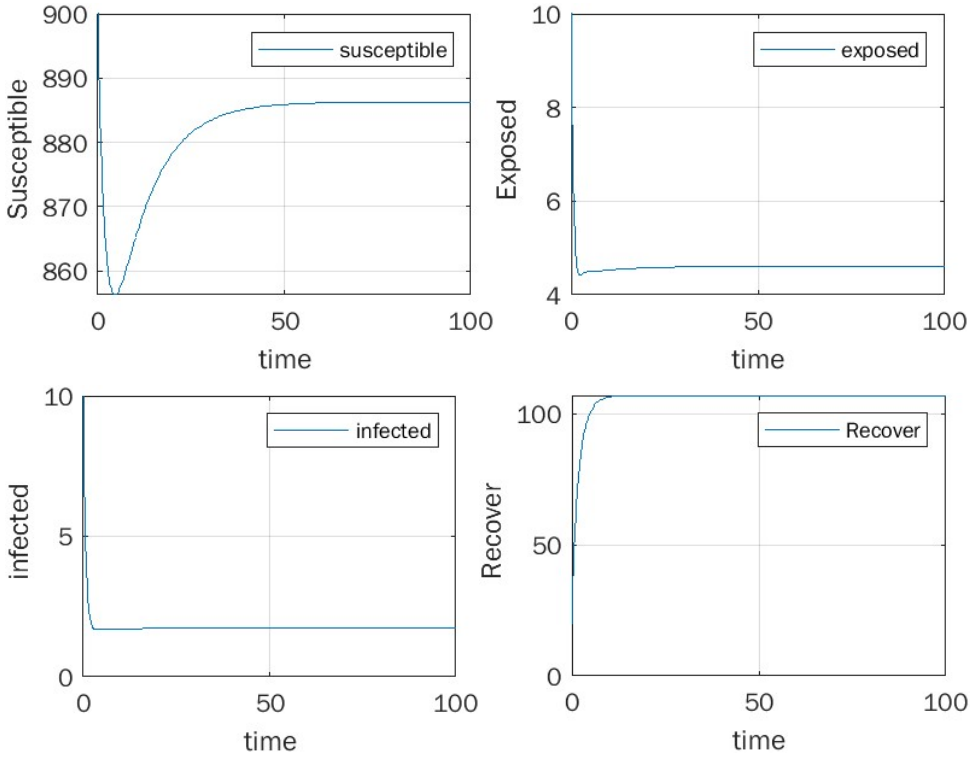

Figure 3: solution curves at endemic with respect to parameter Q_1.

5. CONCLUSION

In this paper we present a SERIS Epidemic model with Holling-type III incidence rate for COVID-19. In this model combine effect of vaccination and treatment is analysis. Dynamical behaviour of our model like positivity, bounded, different equilibriums and sharp threshold parameter reproduction number is investigated. Reproduction number is very much essential to sort out the characterization of the disease. Global stability at Disease-free equilibrium and Endemic equilibrium is investigated the disease will be die-out when $R0$ <1, it became endemic equilibrium is globally asymptotically stable in $R0$ >1 and disease persist. Also analysis for R0 = 1 system has a positive equilibrium exists a when R0 cross one. This emphasizes that the system exhibits transcritical bifurcation at R0 =1.

The sensitivity analysis shows that the most sensitive parameter are contact rate between susceptible and infected (β), transfer rate from exposed to infected (σ) and rate at which recovered loss immunity and go to the susceptible (α). The sensitivity analysis gives a concrete measurement of relative changes in reproduction number to all parameter.

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