STOCHASTIC ANALYSIS OF TWO UNIT COLD STAND BY SYSTEM WITH FAILURE RATE BY HUMAN ERROR

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Abstract: In this paper, we have analyzed a two-unit standby system that can enter a failure state due to human errors. Machines often fail because of various reasons, such as maintenance errors, misinterpretation of the system, or operational mistakes. The system under consideration consists of two identical units – one unit operates actively, while the other is kept in a cold standby mode. When the operative unit fails, the standby unit is immediately activated, ensuring continuous system operation. Simultaneously, the failed unit is sent for repair without delay .In this paper various terms like busy period of the repairman and expected operation time of the system, the expected number of visits by the repairman and expected profit . These performance indicators provide valuable insights into the system's reliability and efficiency, helping to optimize maintenance schedules and minimize operational downtime. These results will be helpful to enhance decision-making processes of system management and allocation of various resources.

KEYWORDS: Two unit cold standby system,Failure unit,Operative unit,Human error,Expected number of visits,MTSF.

1. INTRODUCTION

We have examined two unit systems under the assumption that a unit specialist will arrive with his assistant and begin fixing the malfunctioning unit.If the second unit fails during the first unit's repair period, it is sent to an assistant for repair after receiving instructions, if necessary, and the skilled technician stays on the system all the units are repaired during his stay at the system.There may also be situations when the expert repairman may not devote much time to the system due to busy schedule in other system and hence goes after giving instructions to his assistant for repair of each of the units I'.e the whole system is repaired by the assistant repairman.

provides repair instructions for each unit that malfunctions while he is using the system, then returns and rewords it and today's world, machines and systems are expected to work smoothly and reliably. However, every system can fail at some point, and understanding how and when it might fail helps us plan better and avoid unexpected breakdowns. One way to do this is through **stochastic analysis**—a mathematical method that uses probability to study uncertain events, like system failures.

This paper focuses on a **two-unit cold standby system**. That means we have two machines or units, but only one is working at a time. The second unit is kept as a backup and will only start working if the first one fails. This kind of setup is common in industries, hospitals, and even IT servers where continuous operation is critical.

A new and important part of this study is that it also considers **human error** as a reason for failure. In many real-world systems, human mistakes—like pressing the wrong button or doing poor maintenance—can cause the system to fail. So, to get a more realistic understanding of how the system behaves, we include human error in our analysis.

Using stochastic tools like **Markov processes**, the study looks at how likely the system is to fail over time, how often failures might happen, how long the system can be expected to run before failing, and how human mistakes change these results. This helps engineers and planners improve system design, reduce downtime, and manage risk better.

Expressions for the various reliability characteristics of the system effectiveness, such as mean time to system failure (MTSF), steady state availability of the system, the total fraction of busy time to the expert and his assistant repairman per unit time, and the expected number of visits by the expert repairman, are determined using the regenerative process and semi Markov process technique the above measures.Graphs pertaining to a particular case are also plotted.

2. LITERATURE REVIEW

Cheng et al. (1998) proposed an improved neural network approach for reliability analysis. Their method enhanced the accuracy and efficiency of reliability predictions in microelectronics by optimizing neural network structures for modeling failure behavior.

Agarwal and Renaud (2004) introduced a reliability-based design optimization (RBDO) approach using response surface methodology, particularly suited for multidisciplinary systems. Their work enabled faster convergence by approximating performance functions through surrogate models.

Agarwal and Renaud (2006) presented a **new decoupled RBDO framework**, separating reliability analysis from optimization. This made the computational process more efficient and suitable for complex engineering systems.

Chiu et al. (2006) developed a **genetic algorithm** for task assignment in distributed systems with k-duplications. The method emphasized **reliability optimization** under resource constraints and task replication.

Agarwal et al. (2007) proposed an **inverse-measure-based unilevel architecture** for RBDO. Their technique streamlined optimization by converting the problem into a more tractable single-level format, beneficial for large-scale engineering applications.

Abhilasha et al.(2022) They analyzed **redundant systems subject to degradation during repair**, considering probabilistic models to assess system behavior under partial recovery states.They studied **two-unit warm standby systems** with different failure modes, offering novel insights into system reliability under mixed instructions and varied failure distributions.

Abhilasha et al.(2024) They presented a **reliability model in the context of sustainability and revival**, exploring reliability in environmental and system recovery scenarios.They analyzed **repair rates using different modes of instruction**, contributing to maintenance modeling and optimization.

3. Model Description and Assumptions

1. The system consists of two identical units.Initially one is operative and the other is kept as cold standby.The standby unit cannot fail.
2. Should both units fail, the system becomes unusable.
3. It takes very little time for the skilled repairman to get to the system when he is called in to do the task.
4. Following the expert's instructions, the assistant repairman flawlessly fixes the malfunctioning machine.
5. The expert receives payment for both his visits and the time he spends providing instructions.
6. While the assistant repairman is paid for the time they spend working on repairs, they are not paid for their visits.
7. While the distribution of repair time for the assistant repairman and instruction time are considered generic, the time to failure for each unit has an exponential distribution.
8. Every random variable is independent of every other random variable.

4. Notations:-

$g_1(t)$ =p.d.f of time to repair the failed unit due to some error

$G_1(t)$=c.d.f of time to repair the failed unit due to some error

$f_1(t)$=p.d.f of time to repair the failed unit

$F_1(t)$=c.d.f of time to repair the failed unit

$h_1(t)$=p.d.f of system after repair the unit

$H_1(t)$=c.d.f of system after repair the unit

$E_1(t)$= failure in system due to worker

α= failure rate when unit is in operating state

β= switching rate of the stand by unit

Z_0= when unit is in operating state

Z_s= when unit is in operating state

Z_b= when system is being switched to stand by unit

F_r= after failure,failed unit go under repair

F_R= failed unit is repaired ,and continuing on earlier state

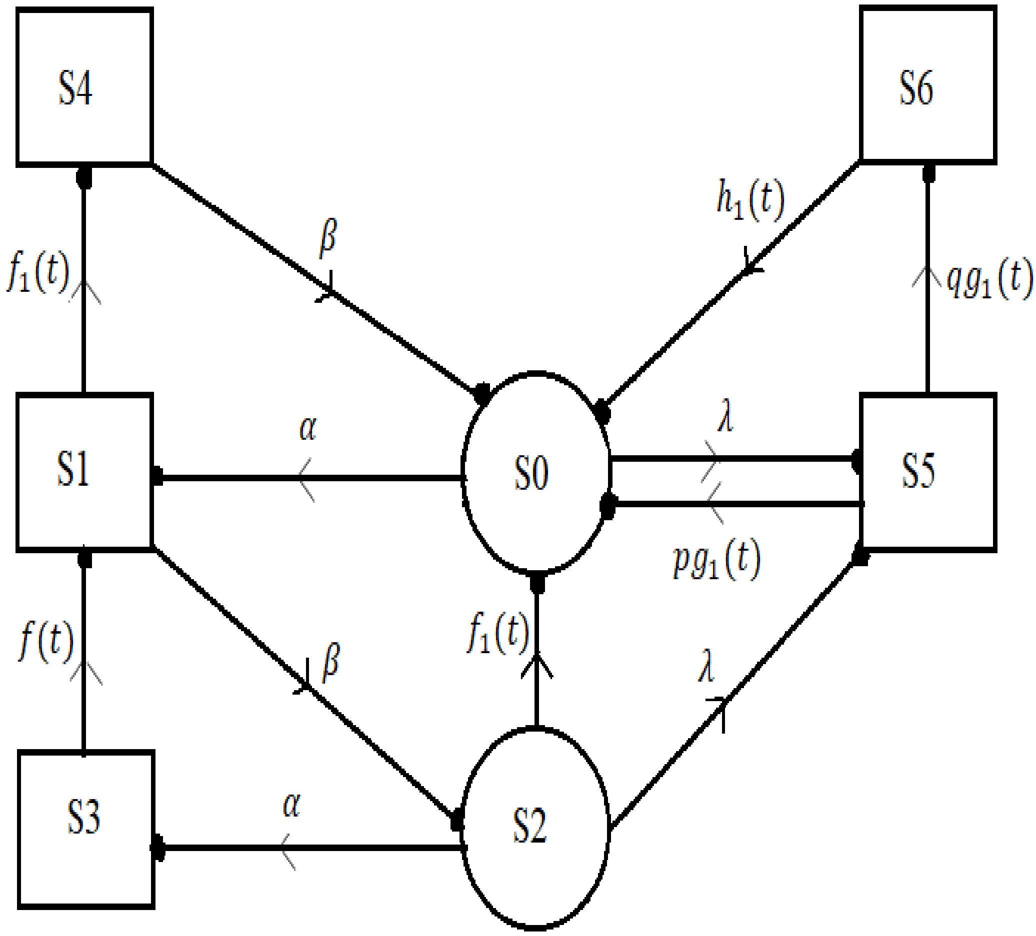
F_w= failed unit is waiting for repair

λ_1= rate at which when both units are in operative state p= probability of failure

q=1−p,

MODEL

In this model, even if a second unit fails while the expert is still in the system, he only provides instructions for fixing one malfunctioning unit at a time. Every time an assistant needs instructions to fix another malfunctioning equipment, he returns to the system. The system's potential states and their transitions are displayed as follows:



5. **<u>Transition Probabilities:-</u>**

$dQ_{01}= \alpha\, e^{-(\alpha+\lambda)t}dt$

$dQ_{05}= \lambda e^{-(\alpha+\lambda)t}dt$

$dQ_{14}=h(t).e^{-\beta t}\, dt$

$dQ_{12}=\beta e^{-\beta t}.\bar{F}_{1(t)dt}$

$dQ_{20}=e^{-(\alpha+\lambda)t}\, .f_1(t)\, dt$

$dQ_{25}=\lambda e^{-(\alpha+\lambda)t}.\bar{F}_{1(t)dt}$

$dQ_{23}=\alpha\, e^{-(\alpha+\lambda)t}.\, \bar{F}1_{(t)dt}$

$dQ_{31}= f_1\, (t)\, dt$

$dQ_{40}=\beta e^{-\beta t}dt$

$dQ_{50}=pg_1(t)dt$

$dQ_{56}=qg_1(t)dt$

$dQ_{60}=h_1(t)dt$ (1.1.01-1.1.12)

The following are the non-zero components of transition probability:

$$P_{ij}= \lim_{s\to 0}\int_0^\infty e^{-st}.dQ_{ij}.\, dt$$

$P_{01}=\frac{\alpha}{\alpha+\lambda}$; $P_{05}=\frac{\lambda}{\alpha+\lambda}$; $P_{14}=f_1^*(\beta)$; $P_{12}=1-f_1^*(\beta)$; $P_{20}=f_1^*(\alpha+\lambda)$; $P_{25}=\frac{\lambda}{\alpha+\lambda}[1-f_1^*(\alpha+\lambda)]$;

$P_{23}=\frac{\alpha}{\alpha+\lambda}[1-f_1^*(\alpha+\lambda)]$; $P_{31}=1$; $P_{40}=1$; $P_{50}=p$; $P_{60}=1$; (1.1.13-1.1.22)

Clearly $P_{01}+P_{05}=1$; $P_{12}+P_{14}=1$; $P_{20}+P_{23}+P_{25}=1$; $P_{31}=P_{40}=P_{60}=1$; $P_{50}+P_{56}=1$ (1.1.23)

When counting from the epoch of entering the state, the system's unconditional mean transit time for each regenerative state 'i' is expressed mathematically as follows:

$M_{ij}=\int t.dQ_{ij}(t)=-\frac{d}{ds}q_{ij}^*(s)|s=0$ (1.1.24)

$M_{01}=\frac{\alpha}{(\alpha+\lambda)^2}$; $M_{05}=\frac{\lambda}{(\alpha+\lambda)^2}$; $M_{14}=-f_1^*(\beta)$; $M_{12}=\frac{1}{\beta}(1-f_1^*(\beta))+f_1^{*'}(\beta)$; $M_{20}=-f_1^{*'}(\alpha+\lambda_2)$; (1.1.25-1.1.29)

$\mu_0=m_{01}+m_{05}=\frac{1}{(\alpha+\lambda)}$

$\mu_1=m_{14}+m_{12}=\frac{1}{\beta}(1-f_1^{*'}(\beta))$

$\mu_2 = m_{20} + m_{23} + m_{25} = \frac{1}{(\alpha+\lambda)}(1 - f_1^*(\alpha+\lambda))$

$\mu_3 = m_{31} = -f_1^{*'}(s)|s=0$

$\mu_4 = m_{40} = \frac{1}{\beta}$

$\mu_5 = m_{50} + m_{56} = -g_1^{*'}(s)|s=0$

$\mu_6 = m_{60} = -h_1^{*'}(s)\ |s=0$ (1.1.30-1.1.36)

5. **System Failure Mean Time (MTSF)**

We consider the failed states 1, 3, 5, and 6 to be observing in order to calculate the "MTSF" for our system.Using probabilistic reasoning, we have:

$\emptyset_0(t) = Q_{01}(t) + Q_{05}(t)$

$\emptyset_2(t) = Q_{23}(t) + Q_{25}(t) + Q_{20}(t)\boxed{s}\emptyset_0(t)$ (1.1.37-1.1.38)

By solving the equations using Laplace Stieltjes' transform, we have

$\emptyset_0^{**}(s)$

$\emptyset_0^{**}(s) = \frac{N(s)}{D(s)} = \frac{Q_{10}^{**}(s) + Q_{05}^{**}(s)}{1}$ (1.1.39)

Considering that the system began at the start of state 0, the MTSF is now,

$T_0 = \lim_{s\to 0} \frac{1 - \emptyset_0^{**}(s)}{s}$ (1.1.40)

Using L' Hospital rule and putting the value of $\emptyset_0^{**}(s)$ we get

$T_0 = \frac{N_1}{D_1} = \mu_0 = \frac{1}{(\alpha+\lambda)}$ (1.1.41)

7.Availability Analysis: $M_i(t)$ – The likelihood that a system that began in a regenerative state would continue to function continuously until time "t" without changing to another regenerative state.

we have :- $M_0 = e^{-(\alpha+\lambda)t}$

$M_2 = e^{-(\alpha+\lambda)t}\bar{F}_1(t)$ (1.1.-42-1.1.43)

$A_0 = M_0 + Q_{01}(c)\ A_1 + Q_{05}(c)\ A_5$

$A_1 = Q_{12}(c)\ A_2 + Q_{14}(c)\ A_4$

$A_2 = M_2 + Q_{20}(c)\ A_0 + Q_{23}(c)\ A_3 + Q_{25}(c)\ A_5$

$A_3 = Q_{31}(c)\ A_1$

$A_4 = Q_{40}(c)\ A_0$

$A_5 = Q_{56}(c)\, A_6 + Q_{50}(c)\, A_0$

$A_6 = Q_{60}(c)\, A_0$ (1.1.44-1.1.50)

$D(s) = -q^*_{60}\ q^*_{01}\ q^*_{12}\ q^*_{25}\ q^*_{56} - q^*_{60}\ q^*_{05}\ q^*_{56} + q^*_{60}\ q^*_{05}\ q_{56}\ q^*_{23}\ q^*_{31}$
$q^*_{12} - q^*_{05}\ q^*_{50} - q^*_{50}\ q^*_{01}\ q^*_{12}\ q^*_{25} + q_{05}\ q^*_{50}\ q^*_{12}\, q^*_{23}\, q^*_{31} - q^*_{01}\, q^*_{40}$
$q^*_{14} - q^*_{12}\, q^*_{23}\, q^*_{31} - q^*_{01}\, q^*_{12}\, q^*_{20} + 1$

$N(s) = M^*_0 - M^*_0\, Q^*_{12}\, Q^*_{23}\, Q^*_{31} + M^*_2\, Q^*_{01}\, Q^*_{12}$

The steady state availability of the system is given by

$$A_0 = \lim_{s\to 0} S\ A^*_0(s) = \lim_{s\to 0} \frac{S\,N(s)}{D(s)} \qquad (1.1.51)$$

$$A^*_0 = \frac{N(s)}{D(s)} \qquad (1.1.52)$$

$$A_0 = \lim_{s\to 0} \frac{S\,N(s)}{D(s)} = \frac{0}{0}\ \text{(form)} \qquad (1.1.53)$$

Now, using L-hospital rule, we get

$A_0 = \frac{N_1}{D_1}$, where $N_1 = \frac{d}{ds} N(s)$ and $D_1 = \frac{d}{ds} D(s)$

Where $N_1 = \mu_0(1 - p_{12}p_{23}) + \mu_2 p_{01} p_{12}$ (1.1.54)

$$D_1 = \mu_0(p_{05} + p_{23}p_{14} + p_{01}p_{20}) + \mu_1 p_{01} + \mu_2 p_{01} p_{12} + \mu_3(p_{12}p_{23} - p_{12}p_{23}p_{05}) + \mu_4 p_{01} + \mu_5 p_{05} + \mu_6 p_{05} p_{50} \qquad (1.1.55)$$

Hence, $A_0 =$

$$= \frac{\mu_0(1 - p_{12}\,p_{23}) + \mu_2\ p_{01}\ p_{12}}{\mu_0(\ p_{05} + p_{23}p_{14} + p_{01}\ p_{20}) + \mu_1\ p_{01} + \mu_2\ p_{01}\ p_{12} + \mu_3\ (p_{12}\ p_{23} - p_{12}\ p_{23}\ p_{05})\ \mu_4\ p_{01} + \mu_5\ p_{05} + \mu_6\ p_{05}\ p_{50}}$$
(1.1.56)

TABLE- 1.1

MTSF T_0 vs. Failure Rate λ				
Sr. No.	λ	**T_0 $p_{12} = 0.75$; $p_{24} = 0.25$**	**T_0 $p_{12} = 0.50$; $p_{24} = 0.50$**	**T_0 $p_{12} = 0.25$; $p_{24} = 0.75$**
1	0.0015	210618.6000	175999.0412	151194.4032
2	0.0025	53186.5842	44519.4931	38299.5368

3	0.0035	23877.4189	20019.9904	17249.0615
4	0.0045	13566.8603	11394.0937	9831.2968
5	0.0055	8760.6767	7378.1918	6375.3793
6	0.0065	6151.4250	5184.1206	4485.8974
7	0.0075	4565.9767	3853.5878	3339.2766
8	0.0085	3531.2590	2985.1127	2590.3239
9	0.0095	2818.4272	2386.3200	2073.5976

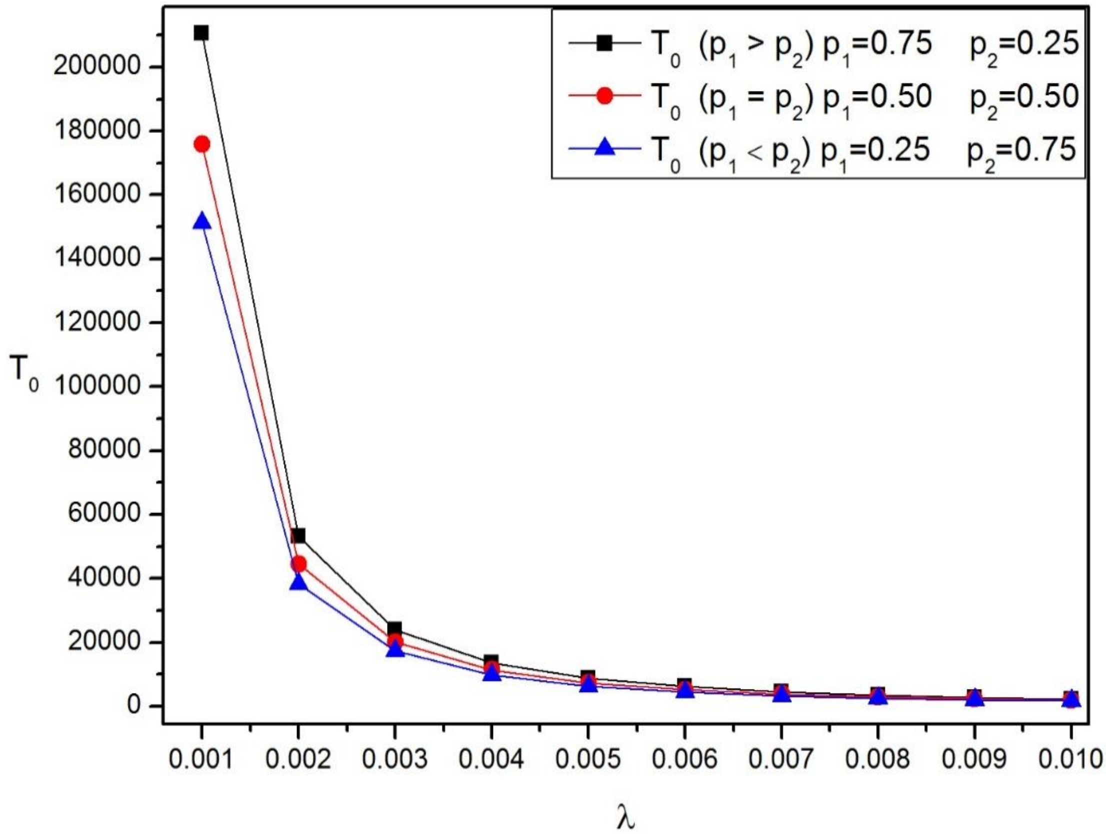


Figure 1 (MTSF T_0 vs. Failure Rate of the system)

8.COMPARISON ANALYSIS

MTSF vs. Failure Rate of the Main Unit with Initial State S1

The MTSF has been evaluated for the different operating unit failure rate values (λ) as shown in Table 1.2, taking into account the different scenarios of likelihood of repair (p12) and replacement (p24) that the system requires. Figure 2 shows the related graphs for these cases.

Table 1.2 and Figure 2 show that when the operational unit's failure rate (λ) rises, the system's MTSF (T1) rapidly decreases for any fixed value of p12/p24. The observed range of the percentage decline in MTSF is from 74.85% This percentage drop in T1 is almost the same in all three cases, going down to 18.51% when λ varies between 0.0015 and 0.0095. On the other hand, T1 decreases as p12 decreases or p24 rises for a given value of λ. When p12 falls from 0.75 to 0.25, the value of T1 decreases by 27.54% for λ = 0.0095 and by 28.35% for λ = 0.0015. This suggests that with higher failure rates (λ), the MTSF (T1) variations are less for a given change in p12.

TABLE- 1.2

MTSF T_1 vs. Failure Rate λ				
Sr. No.	λ	**T_1 $p_{12} = 0.75$; $p_{24} = 0.25$**	**T_1 $p_{12} = 0.50$; $p_{24} = 0.50$**	**T_1 $p_{12} = 0.25$; $p_{24} = 0.75$**
1	0.0015	209909.1562	175267.5625	150438.1718
2	0.0025	52887.3281	44193.6640	37973.8281
3	0.0035	23711.5332	19843.0390	16964.1777
4	0.0045	13467.6542	11286.5927	9617.8310
5	0.0055	8611.4794	7212.3569	6204.7636
6	0.0065	6019.9096	5046.0620	4343.8491
7	0.0075	4452.4931	3735.3701	3317.6325
8	0.0085	3432.0729	2881.7756	2483.9848
9	0.0095	2730.3559	2294.5664	1979.1617

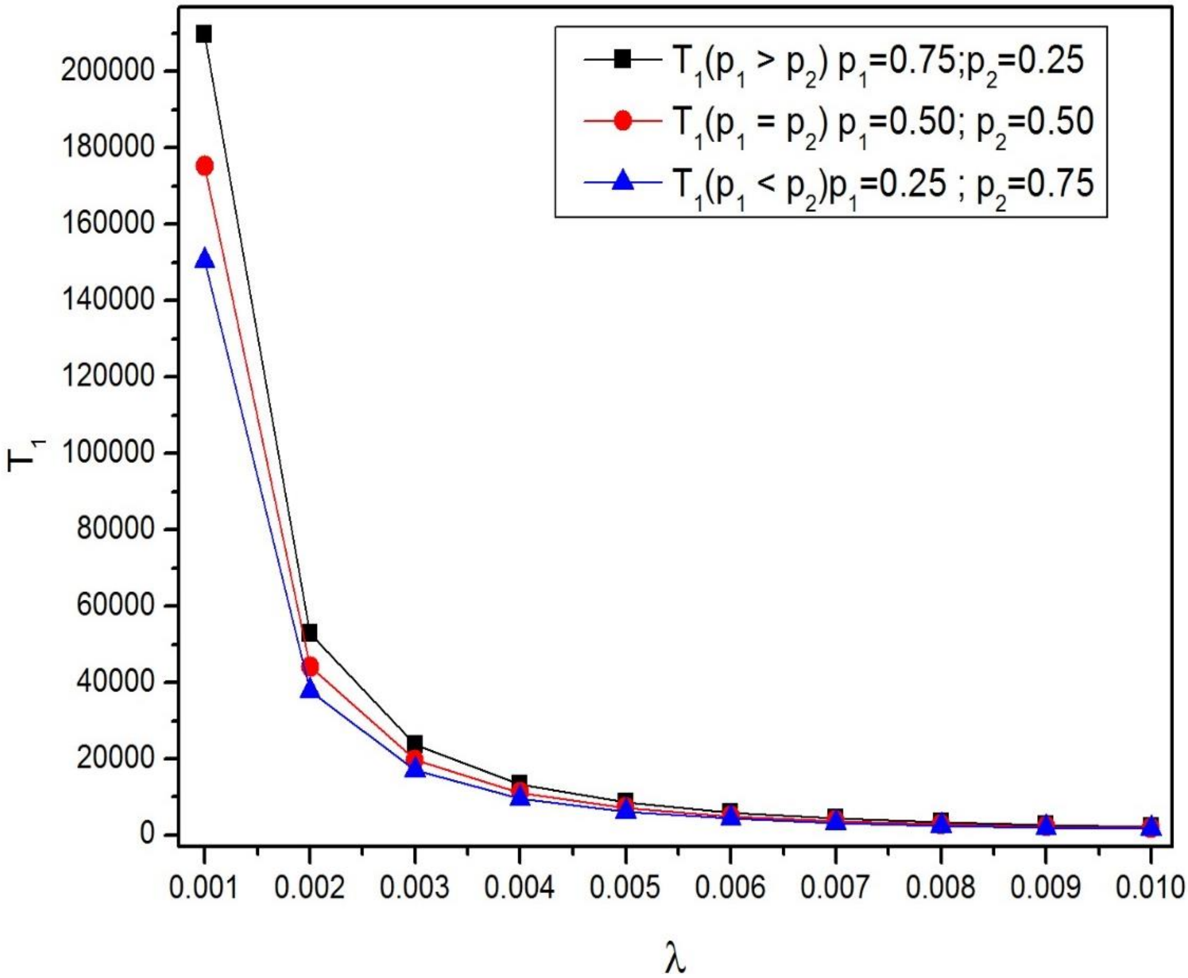


Figure 2 (MTSF T_1 vs. Failure Rate λ of the Main Unit)

9. Conclusion

The Mean Time to System Failure (MTSF) and the availability Path analysis was used to rapidly and efficiently design the two-unit cold standby system. It has been established that MTSF is affected by the starting state since it is regarded as a positional measure. On the other hand, the system's steady state availability, which serves as a global metric, has been shown to be the same even if it was computed independently using S0 and S1 as the beginning states. Furthermore, the system analysis revealed that almost all MTSFs pertaining to S0 and S1 (as beginning states) are relevant and Given any fixed level of minor or major failure probabilities and the rates of inspection, replacement, and repair, the probability of successful operation rapidly decreases as the operational unit's failure rate increases.

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