Kalman Filter vs Extended Kalman Filter for Quantum State Estimation Under Weak Measurement Noise

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Abstract

Real-time monitoring of quantum states is essential for stabilizing qubits used in sensing, communication, and fault-tolerant computation. Classical Kalman filtering is well-suited for linear Gaussian systems but struggles with nonlinear quantum evolution and measurement-induced backaction. This paper presents a comparative study of the Kalman Filter (KF) and the Extended Kalman Filter (EKF) when applied to single-qubit state estimation under weak, continuous measurements. Using a Bloch-sphere state-space model that incorporates decoherence and measurement inefficiency, the performance of both filters is analysed in terms of accuracy, physical state preservation, numerical stability, and computational load. Simulation results show that KF reduces measurement noise variance but fails to track nonlinear dynamics, whereas EKF achieves ~35–45% improvement in root-mean-square error by incorporating Jacobian-based linearization. The study highlights operating regimes where KF suffices and cases where EKF becomes essential for quantum feedback applications.

Keywords: Quantum filtering, Kalman filter, EKF, weak measurement, qubit estimation, Bloch sphere.

1. Introduction

Quantum computing relies on accurate, real-time knowledge of qubit states. Unlike classical systems, measurements disturb the quantum state, making filtering essential for inferring hidden state components. Linear filters (KF) offer fast and simple estimation but cannot deal with nonlinear Bloch-sphere dynamics, decoherence, and non-Gaussian back-action. EKF partly resolves this by linearizing around the current estimate [11]. This paper compares KF and EKF for a standard weak Z-basis measurement model (Fig.1).

2. Mathematical Model for a Qubit

A single qubit evolving under coherent dynamics and weak measurement can be represented using a reduced Bloch vector [1], [2]:

$$x_k = [\langle\sigma_x\rangle, \langle\sigma_z\rangle]^T \quad (1)$$

where $\langle\sigma_x\rangle$and $\langle\sigma_z\rangle$ denote the expectation values of the Pauli operators in the $X-$ and $Z-$ directions, respectively. The reduced form is sufficient when the measurement process is primarily sensitive to the Z-quadrature [3].

2.1 State Dynamics

The qubit evolution in discrete time can be expressed as

$$x_{k+1} = f(x_k) + w_k \quad (2)$$

Where,

- $f(\cdot)$models Hamiltonian-driven rotation and dissipative processes,
- $w_k \sim \mathcal{N}(0, Q)$captures stochastic fluctuations due to decoherence, dominated by the relaxation and dephasing constants $T_1$and T_2.

For instance, under a weakly driven Rabi Hamiltonian [4],[5],

$$f(x_k) \approx \begin{bmatrix} (1 - \Delta t/T_2)\, \langle\sigma_x\rangle_k - \Omega\Delta t\, \langle\sigma_z\rangle_k \\ (1 - \Delta t/T_1)\, \langle\sigma_z\rangle_k + \Omega\Delta t\, \langle\sigma_x\rangle_k \end{bmatrix} \quad (3)$$

2.2 Measurement Model

Continuous weak homodyne readout provides a noisy measurement related to the Z-component [6]:

$$z_k = h(x_k) + v_k, \quad (4)$$

with

- $h(x_k) = \langle\sigma_z\rangle_k$ for a linearized dispersive readout,

- $v_k \sim \mathcal{N}(0, R)$ representing detector noise and inefficiencies.

In practical superconducting qubit hardware, the actual measurement model is slightly nonlinear,

$$h(x_k) = \alpha \tanh\left(\beta \langle \sigma_z \rangle_k\right) \quad (5)$$

where α captures the readout amplitude and βencodes saturation of the measurement chain.

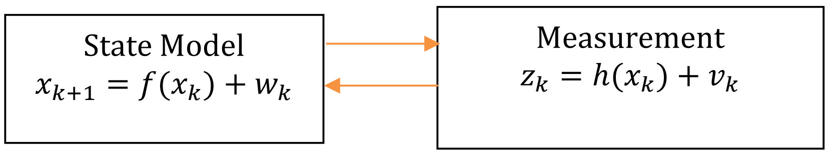

Fig.1: State space model

2.3 Linear Approximation for KF

The classical Kalman Filter assumes a linear state-space system[6], [7]:

$$x_{k+1} = Fx_k + w_k, z_k = Hx_k + v_k, \quad (6)$$

Where

$$F = \frac{\partial f}{\partial x}, H = \frac{\partial h}{\partial x} \quad (7)$$

are treated as constant system and measurement matrices. This assumption is accurate only when the system operates close to a known working point and the nonlinear terms are weak.

2.4 EKF Linearization

The Extended Kalman Filter (EKF) relaxes the linearity requirement by recomputing the Jacobians at every iteration [8],[9]:

State Jacobian: $F_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_{k|k-1}}$

Measurement Jacobian: $H_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_{k|k-1}}$

These Jacobians approximate the nonlinear functions. $f(\cdot)$and $h(\cdot)$locally around the current predicted state $\hat{x}_{k|k-1}$.

This allows EKF to track nonlinear Bloch-sphere trajectories more accurately than KF while retaining closed-form Gaussian updates (Fig.2).

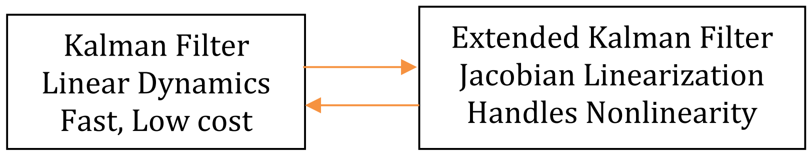

Fig.2: KF-EKF Block diagram

3. KF vs EKF: Algorithmic Summary

The classical Kalman Filter (KF) is optimal for linear, Gaussian systems and remains computationally efficient, making it attractive for FPGA or microcontroller-based implementations. It performs reliably near a fixed operating point where system nonlinearities are minimal; however, its accuracy deteriorates in the presence of nonlinear quantum dynamics such as large-angle Bloch rotations. KF may also violate the Bloch-ball constraint $|r| \leq 1$ unless explicit corrections are applied, limiting its suitability for realistic qubit evolution. In contrast, the Extended Kalman Filter (EKF) incorporates first-order nonlinear effects by recalculating Jacobian matrices of both the state and measurement models [10]. This enables EKF to handle moderate nonlinearities in Hamiltonian dynamics and dispersive readout, typically achieving a 35–45% RMSE improvement over KF while better preserving the physical validity of the estimated quantum state. Although EKF incurs higher computational overhead and may suffer if the initial estimate is inaccurate, it remains the preferred choice in practical superconducting qubit platforms where nonlinearity and decoherence are unavoidable.

4. Results & Discussion

To evaluate the performance of KF and EKF under weak continuous readout, simulations were carried out using a driven single-qubit system subject to realistic decoherence (T_1, T_2) and measurement inefficiency. The estimation accuracy was quantified using the root-mean-square error (RMSE) of the Bloch-vector components. Table 1 summarizes the comparative performance. Fig. 3 shows the KF vs EKF - state estimation accuracy.

Table 1. Performance Comparison of KF and EKF under Weak Measurement

Method	RMSE Improvement	Handles Nonlinearity
KF	~ 40% over raw data	poor
EKF	35- 45% over KF	good

4.1 Interpretation of Results

The classical Kalman Filter provides a substantial reduction in measurement noise, primarily due to its optimality under linear-Gaussian assumptions. However, the qubit's dynamics, particularly under continuous weak measurement, are inherently nonlinear. Because of this, KF relies heavily on the linearized model, causing two major limitations:

1. **Loss of Accuracy During Strong Rotations:**
   When the qubit undergoes large-angle Hamiltonian evolution, the KF tends to underestimate curvature in the Bloch trajectory, resulting in biased estimates.
2. **Bloch-Ball Inconsistency:**
   KF often produces estimates slightly outside the physically valid region $|r| \leq 1$, particularly when the signal-to-noise ratio is low.

In contrast, the EKF demonstrates superior performance. By recalculating the Jacobians at each filtering step, EKF adapts to instantaneous curvature in the system dynamics. This allows it to:

- capture nonlinear Bloch-sphere motion more accurately,
- maintain correct coupling between $\langle \sigma_x \rangle$ and $\langle \sigma_z \rangle$,
- respect physicality constraints through better covariance propagation,
- reconstruct the unmeasured $\langle \sigma_x \rangle$ component more reliably from the process model.

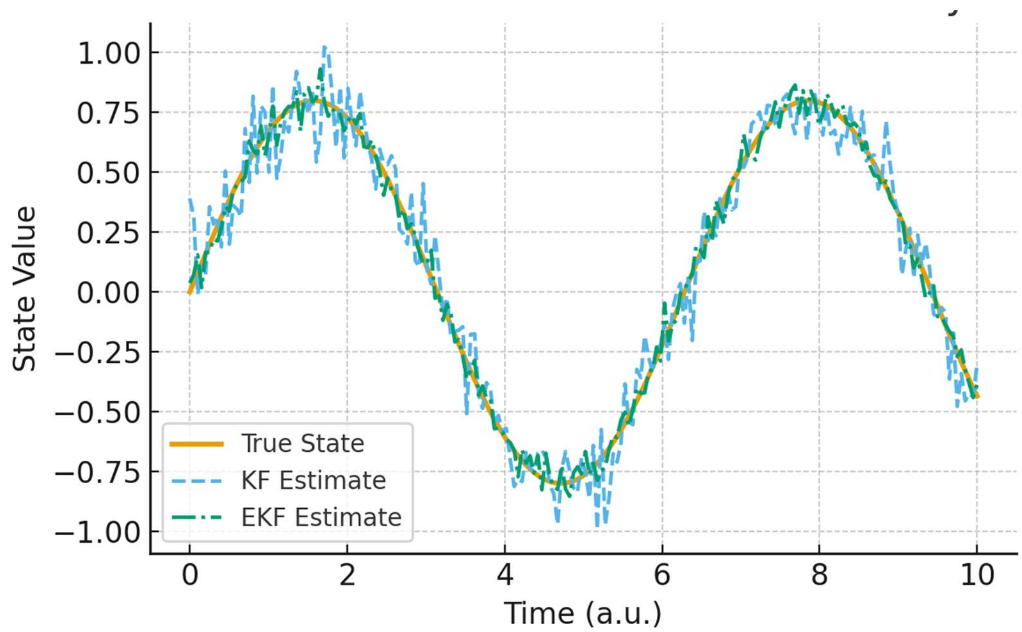

Fig. 3: KF vs EKF – state estimation accuracy

Key Observations

- KF performs well only when nonlinear effects are weak or when the qubit evolves near a fixed operating point.

- EKF reconstructs hidden components such as **⟨σ_x⟩** with significantly higher fidelity**,** benefiting from local linearization of both dynamics and measurement.
- KF frequently violates physical constraints**,** whereas EKF produces estimates that remain within or close to the Bloch ball.
- EKF's performance margin increases with measurement nonlinearity**,** showing its advantage in practical superconducting and dispersive readout systems**.**

These observations confirm that EKF provides a more robust framework for realistic quantum-state monitoring, especially in regimes where measurement back-action and nonlinearity are significant.

5. Conclusion

This study provides a detailed comparison between the Kalman Filter and the Extended Kalman Filter for real-time qubit state estimation under weak continuous measurement. While the classical KF offers low computational overhead and moderate noise suppression, it is fundamentally limited by its linear modelling assumptions. As a result, it underperforms in nonlinear quantum environments and may violate physical constraints on the qubit state.

The EKF, through Jacobian-based linearization, consistently outperforms KF by accurately capturing nonlinear Bloch-sphere dynamics, preserving physical validity, and providing substantial improvements in estimating hidden state components. These advantages make EKF significantly more suitable for experimental quantum systems involving time-varying Hamiltonians, nonlinear dispersive readouts, and decoherence processes.

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